IIT-JEE / ENGINEERING

MATHS

Assignment 1.0

VECTOR

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1. If $\vec{a} \times \vec{b} = \vec{c} \times \vec{d}$ and $\vec{a} \times \vec{c} = \vec{b} \times \vec{d}$, prove that $\vec{a} - \vec{d}$ is parallel to $\vec{b} - \vec{c}$ provided $\vec{a} \neq \vec{d}$ and $\vec{b} \neq \vec{c}$.
2. If a, b, c are the lengths of the sides opposites respectively to the angles A, B and C of a ΔABC, using vectors. Prove that $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$.
3. Define $\vec{a} \times \vec{b}$ and prove that $|\vec{a} \times \vec{b}| = (\vec{a}.\vec{b}) \tan\theta$, where θ is the angle between the vectors $\vec{a}$ and $\vec{b}$.
4. Using vectors, prove that the altitudes of a triangle are concurrent.
5. Find the work done in moving a particle from the point A, with position vector $2\hat{i} - 6\hat{j} + 7\hat{k}$, to the point B, with position vector $3\hat{i} - \hat{j} + 5\hat{k}$ by a force $\vec{F} = \hat{i} + 3\hat{j} - \hat{k}$.
6. Find a unit vector perpendicular to both $\vec{a} = 3\hat{j} + \hat{j} - 2\hat{k}$ and $\vec{b} = 2\hat{i} + 3\hat{j} - \hat{k}$.
7. Find λ if $\vec{a} = 4\hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = \lambda\hat{i} - 2\hat{j} + 2\hat{k}$ are perpendicular to each other.
8. Find a unit vector perpendicular to both $\vec{a} = 2\hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = 3\hat{i} - \hat{j} + \hat{k}$.
9. Find λ if the vector $\vec{a} = 3\hat{i} + \hat{j} - 2\hat{k}$ and $\vec{b} = \hat{i} + \lambda\hat{j} - 3\hat{k}$ are perpendicular to each other.
10. Find a unit vector perpendicular to both the vectors $\vec{a} = 4\hat{i} - \hat{j} - 3\hat{k}$ and $\vec{b} = 2\hat{i} + 2\hat{j} - \hat{k}$
11. Find λ if the vector $\vec{a} = \hat{i} - \lambda\hat{j} + 3\hat{k}$ and $\vec{b} = 4\hat{i} - 5\hat{j} + 2\hat{k}$ are perpendicular to each other.
12. Find a unit vector perpendicular to both the vectors $\vec{a} = 4\hat{i} + 2\hat{j} - \hat{k}$ and $\vec{b} = \hat{i} + 4\hat{j} - \hat{k}$.
13. Find λ if the vector $\vec{a} = \hat{i} - 3\hat{j} + 3\hat{k}$ and $\vec{b} = \lambda\hat{i} - \hat{j} + 2\hat{k}$ are perpendicular to each other.
14. Find a unit vector in the direction of $(\vec{a} + \vec{b})$ where $\vec{a} = -\hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} + \hat{j} - 3\hat{k}$.
15. Find a unit vector in the direction of $(\vec{a} - \vec{b})$ where $\vec{a} = -\hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} + \hat{j} - 3\hat{k}$.
16. Find a unit vector in the direction of $(\vec{a} + \vec{b})$ where $\vec{a} = \hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + 3\hat{k}$.
17. Find the projection of $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$ on $\vec{b} = \hat{i} - 2\hat{j} + \hat{k}$.
18. Find a vector whose magnitude in 3 units and which is perpendicular to the following two vectors $\vec{a}$ and $\vec{b}$: $\vec{a} = 3\hat{i} + \hat{j} - 4\hat{k}$ and $\vec{b} = 6\hat{i} + 5\hat{j} - 2\hat{k}$.
19. Using vectors prove that the right bisector of the sides of a triangle are concurrent.

20. If $\vec{a}=4\hat{i}+3\hat{j}+\hat{k}$ and $\vec{b}=\hat{i}-2\hat{k}$, find $2\vec{b}$ x $\vec{a}$.
21. If $\vec{a}=4\hat{i}+3\hat{j}+2\hat{k}$ and $\vec{b}=3\hat{i}+2\hat{k}$, find $\vec{b}$ x $2\vec{a}$.
22. If $\vec{a}=\hat{i}+\hat{j}-3\hat{k}$ and $\vec{b}=\hat{j}+2\hat{k}$, find $|2\vec{b} \text{ x } \vec{a}|$.
23. If three vectors $\vec{a}$, $\vec{b}$, $\vec{c}$ are such that $\vec{a}+\vec{b}+\vec{c}=0$ prove that $\vec{a} \text{ x } \vec{b}=\vec{b} \text{ x } \vec{c}=\vec{c} \text{ x } \vec{a}$.
24. If $\vec{a}=\hat{i}-2\hat{j}+3\hat{k}$ and $\vec{b}=\hat{i}-3\hat{k}$, find $|\vec{b} \text{ x } 2\vec{a}|$.
25. Using vectors, prove that a parallelogram whose diagonals are equal is a rectangle.
26. Using vectors, prove that the diagonals of a rhombus are at right angles.
27. Using vectors, prove that the square of the hypotenuse of a right angled triangle is equal to the sum of the square of other two sides.
28. If $\vec{a}=\hat{i}+\hat{j}+2\hat{k}$ and $\vec{b}=3\hat{i}+2\hat{j}-\hat{k}$, find $(\vec{a}+3\vec{b}).(2\vec{a}-\vec{b})$.
29. If $\vec{a}=\hat{i}+2\hat{j}-3\hat{k}$ and $\vec{b}=3\hat{i}-\hat{j}+2\hat{k}$, show that $(\vec{a}+\vec{b})$ is perpendicular to $(\vec{a}-\vec{b})$.
30. If $|\vec{a}|=5, |\vec{b}|=13$ and $\vec{a} \text{ x } \vec{b}=25$, find $\vec{a}\,.\,\vec{b}$.
31. For any two vectors $\vec{a}$ and $\vec{b}$ show that $(1+|\vec{a}|^2)(1+|\vec{b}|^2)=(1-\vec{a}\,.\,\vec{b})^2+|\vec{a}+\vec{b}+\vec{a} \text{ x } \vec{b}|^2$.
32. If $|\vec{a}|=\sqrt{26}$, $|\vec{b}|=7$ and $|\vec{a} \text{ x } \vec{b}|=35$, find $\vec{a}\,.\,\vec{b}$.
33. Find the area of a parallelogram whose adjacent sides are given by the vectors $\hat{i}-3\hat{j}+\hat{k}$ and $\hat{i}+\hat{j}+\hat{k}$.
34. Prove that: $(\vec{a} \text{ x } \vec{b})^2=|\vec{a}|^2.|\vec{b}|^2-(\vec{a}\,.\,\vec{b})^2$.
35. If a, b, c are respectively the lengths of the sides opposite to the angles A, B, C of a triangle ABC, show that $a=b\cos C+c\cos B$.
36. In ΔOAB $\overline{OA}=3\hat{i}+2\hat{j}-\hat{k}$ and $\overline{OB}=\hat{i}+3\hat{j}+\hat{k}$. Find the area of triangle.
37. The dot product of a vector with the vectors $\hat{i}+\hat{j}-3\hat{k}$, $\hat{i}+3\hat{j}-2\hat{k}$ and $2\hat{i}+\hat{j}+4\hat{k}$ are 0, 5 and 8 respectively. Find the vector.
38. Using vectors, prove that the mid-point of the hypotenuse of a right-angled triangle is equidistant from its vertices.
39. Find the value of λ so that the two vectors $2\hat{i}+3\hat{j}-\hat{k}$ and $4\hat{i}+6\hat{j}+\lambda\hat{k}$ are (i) parallel, (ii) perpendicular to each other.
40. Find the values of λ so that the vectors $2\hat{i}-4\hat{j}+\hat{k}$ and $4\hat{i}-8\hat{j}+\lambda\hat{k}$ are (i) parallel, (ii) perpendicular to each other.
41. Using vectors, prove that angle in a semicircle is a right angle.
42. Find the values of λ so that the vectors $2\hat{i}+4\hat{j}-2\hat{k}$ and $3\hat{i}+6\hat{j}+\lambda\hat{k}$ are (i) parallel, (ii) perpendicular to each other.

43. If $5\hat{i}-\hat{j}-3\hat{k}$ and $\hat{i}+3\hat{j}-5\hat{k}$, then show that the vectors $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$ are orthogonal.
44. Show that the area of a parallelogram having diagonals $3\hat{i}+\hat{j}-2\hat{k}$ and $\hat{i}-3\hat{j}+4\hat{k}$ sq. units.
45. A particle acted on by constant forces $4\hat{i}+\hat{j}-3\hat{k}$ and $3\hat{i}+\hat{j}-\hat{k}$ is displaced from the point $\hat{i}+2\hat{j}+3\hat{k}$ to the point $5\hat{i}+4\hat{j}+\hat{k}$. Find the total work done by the forces.
46. Using vectors, prove that if the diagonals of a parallelogram are equal in length, then it is a rectangle.
47. Express the vector $\vec{a}=5\hat{i}-2\hat{j}+5\hat{k}$ as sum of two vectors such that one is parallel to the vector $\vec{b}=3\hat{i}+\hat{k}$ and the other is perpendicular to $\vec{b}$.
48. Forces $2\hat{i}+5\hat{j}+6\hat{k}$ *and* $-\hat{i}+2\hat{j}-\hat{k}$ act on a particles. Determine the work done when the particle is displaced from A (4, -3, -2) to B(6, 1, -3).
49. If $\vec{a}, \vec{b}$ *and* $\vec{c}$ are three mutually perpendicular vectors of equal magnitude, find the angle between $\vec{a}$ and $(\vec{a}+\vec{b}+\vec{c})$.
50. Let $\vec{a}=\hat{i}-\hat{j}$, $\vec{b}=3\hat{j}-\hat{k}$ and $\vec{c}=7\hat{i}-\hat{k}$. Find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$, and $\vec{c}.\vec{d}=1$.
51. Using vectors, prove that the diagonals of a rhombus are perpendicular bisectors of each other.
52. Using vectors, prove that the median to the base of an isosceles triangle is perpendicular to the base.
53. Show that the point whose position vectors are $\vec{a}=4\hat{i}-3\hat{j}+\hat{k}$, $\vec{b}=2\hat{i}-4\hat{j}+5\hat{k}$ and $\vec{c}=\hat{i}-\hat{j}$ form a right angled triangle.
54. Using vectors prove that if two medians of a triangle ABC be equal then triangle is isosceles.
55. Using vectors prove that the quadrilateral formed by joining the mid points of the adjacent sides of a rectangle is a rhombus.
56. Using vectors prove that the line segment joining the mid points of non parallel sides of a trapezium is parallel to the base and is equal to half the sum of the parallel sides.
57. Find the angle between the vectors $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$ if $\vec{a}=2\hat{i}-\hat{j}+3\hat{k}$ and $\vec{b}=3\hat{i}+\hat{j}-2\hat{k}$.
58. Using vectors, prove that in a $\Delta ABC, \dfrac{a}{\sin A}=\dfrac{b}{\sin B}=\dfrac{c}{\sin C}$, where a, b and c are length of the sides opposites respectively to the angles A, B and C of ΔABC.
59. If $\vec{a}=4\hat{i}+2\hat{j}-\hat{k}$ and $\vec{b}=5\hat{i}+2\hat{j}-3\hat{k}$ find the angle between the vectors $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$.

60. Using vectors, prove that the median to the base of an isosceles triangle is perpendicular to the base.
61. Show that the vectors $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$ are perpendicular to each other for $\vec{a}=\hat{i}-2\hat{j}+3\hat{k}$ and $\vec{b}=3\hat{i}+\hat{j}-2\hat{k}$.
62. Find the value of λ which makes the vectors $\vec{a}, \vec{b}, \vec{c}$ coplanar, where $\vec{a}=2\hat{i}-\hat{j}+\hat{k}$, $\vec{b}=\hat{i}+2\hat{j}-3\hat{k}$ and $\vec{c}=3\hat{i}-\lambda\hat{j}$.
63. Find the projection of $\vec{b}+\vec{c}$ on $\vec{a}$ where $\vec{a}=\hat{i}+2\hat{j}+\hat{k}$, $\vec{b}=\hat{i}+3\hat{j}+\hat{k}$ and $\vec{c}=\hat{i}+\hat{k}$.
64. Find the value of λ which makes the vectors $\vec{a}, \vec{b}, \vec{c}$ coplanar, where $\vec{a}=-4\hat{i}-6\hat{j}-2\hat{k}$, $\vec{b}=-\hat{i}+4\hat{j}+3\hat{k}$ and $\vec{c}=8\hat{i}-\hat{j}+\lambda\hat{k}$.
65. If $\vec{a}=3\hat{i}-\hat{j}+2\hat{k}$ and $\vec{b}=-\hat{i}-2\hat{j}+4\hat{k}$, find a unit vector along the vector $(\vec{a} \text{ x } \vec{b})$.
66. If $\vec{a}=3\hat{i}-4\hat{j}-3\hat{k}$ and $\vec{b}=2\hat{i}-3\hat{k}$, find a unit vector along the vector $(\vec{a} \text{ x } \vec{b})$.
67. Find the projection of $\vec{b}+\vec{c}$ on $\vec{a}$ where $\vec{a}=2\hat{i}-2\hat{j}+\hat{k}$, $\vec{b}=\hat{i}+2\hat{j}-2\hat{k}$ and $\vec{c}=2\hat{i}-\hat{j}+4\hat{k}$.
68. If $\vec{\alpha}=\hat{i}+\hat{j}+\hat{k}$, $\vec{\gamma}=\hat{j}-\hat{k}$, find the vector $\vec{\beta}$ such that $\vec{\alpha} \text{ x } \vec{\beta}=\vec{\gamma}$ and $\vec{\alpha}.\vec{\beta}=3$.
69. If $|\vec{a}|=2, |\vec{b}|=\sqrt{3}$ and $\vec{a}.\vec{b}=\sqrt{3}$, find the angle between $\vec{a}$ and $\vec{b}$.
70. If $\vec{a}=\hat{i}+2\hat{j}-3\hat{k}$ and $\vec{b}=2\hat{i}+4\hat{j}+9\hat{k}$, find a unit vector parallel to $\vec{a}+\vec{b}$.
71. Show that the area of the parallelogram having diagonals $(3\hat{i}+\hat{j}-2\hat{k})$ and $(\hat{i}-3\hat{j}+4\hat{k})$ is $5\sqrt{3}$ sq. units.
72. If $|\vec{a}|=\sqrt{3}, |\vec{b}|=2$ and angle between $\vec{a}$ *and* $\vec{b}$ is 60^0, find $\vec{a}.\vec{b}$.
73. Find a vector in the direction of vector $\vec{a}=\hat{i}-2\hat{j}$, whose magnitude is 7.
74. If $\hat{i}+\hat{j}+\hat{k}, 2i+5\hat{j}, 3\hat{i}+2\hat{j}-3\hat{k}$ and $\hat{i}-6\hat{j}-\hat{k}$ are the position vectors of the points A, B, C and D, find the angle between $\overrightarrow{AB}$ and $\overrightarrow{CD}$. Deduce that $\overrightarrow{AB}$ and $\overrightarrow{CD}$ are collinear.
75. Find a unit vector in the direction of $\vec{a}=3\hat{i}-2\hat{j}+6\hat{k}$.
76. Find the angle between the vectors $\vec{a}=\hat{i}-\hat{j}+\hat{k}$ and $\vec{b}=\hat{i}+\hat{j}-\hat{k}$.
77. For what value of λ are the vector $\vec{a}=2\hat{i}+\lambda\hat{j}+\hat{k}$ and $\vec{b}=\hat{i}-2\hat{j}+3\hat{k}$ perpendicular to each other?
78. If $\vec{a}=\hat{i}+\hat{j}+\hat{k}$ and $\vec{b}=\hat{j}-\hat{k}$, find a vector $\vec{c}$ such that $\vec{a} \text{ x } \vec{c}=\vec{b}$ and $\vec{a}.\vec{c}=3$.
79. If $\vec{a}+\vec{b}+\vec{c}=0$ and $|\vec{a}|=3, |\vec{b}|=5$ and $|\vec{c}|=7$, show that the angle between $\vec{a}$ and $\vec{b}$ is 60^0.

Note : if any mistake on this, kindly inform on the mail id : bknal1207@gmail.com

Your Observation! Our Correction !!