IIT-JEE / ENGINEERING

MATHS

Assignment 1.0

Three Dimensional Geometry

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1. A variable plane passes through a fixed point (1, 2, 3). Show that the locus of the foot of the perpendicular drawn from origin to this plane is the sphere given by the equation $x^2+y^2+z^2-x-2y-3z=0$.
2. Define the line of shortest distance between two skew lines. Find the shortest distance and the vector $\vec{r}=(2\hat{j}-3\hat{k})+\lambda(2\hat{i}-\hat{j})$ *and* $\vec{r}=(4\hat{i}+3\hat{j})+\mu(3\hat{i}+\hat{j}+\hat{k})$.
3. A variable plane passes though a fixed point (2, -1, 5). Show that the locus of the foot of the perpendicular drawn from origin to this plane is the sphere given by the equation $x^2+y^2+z^2-2x+y-5z=0$.
4. A plane passes through a fixed point (1, -2, 3) and cuts the axes in A, B and C. Show that the locus of the centre of the sphere, passing through the points O, A, B and C, is given by $\frac{1}{x}-\frac{2}{y}+\frac{3}{z}=2$.
5. Define the line of shortest distance between two skew lines. Find the shortest distance and the vector equation of the line of shortest distance between the lines given by : $\vec{r}=6\hat{i}+3\hat{k}+\lambda(2\hat{i}-\hat{j}+4\hat{k})$ and $\vec{r}=-9\hat{i}+\hat{j}-10\hat{k}+\mu(4\hat{i}+\hat{j}+6\hat{k})$
6. A plane passes through a fixed point (2, 5, -4) and cuts the axes in A, B and C. Show that the locus of the centre of the sphere, passing through the points O, A, B and C, is given by $\frac{2}{x}+\frac{5}{y}-\frac{4}{z}=2$.
7. A (0, 6, -9), B(-3, -6,3) and C(7, 4, -1) are three points. Find the equation of line AB. If D is the foot of the perpendicular drawn from the point C to the line AB, find the coordinates of the point D.
8. A (-3, -2, -1), B(3, 4,2) and C(4, 1, -3) are three points. Find the equation of line AB. If D is the foot of the perpendicular drawn from the point C to the line AB, find the coordinates of the point D.
9. Find the foot of the perpendicular from (0, 2, 7) on the line $\frac{x+2}{-1}=\frac{y-1}{3}=\frac{z-3}{-2}$.
10. Find the Cartesian as well as vector equations of the planes through the intersection of the planes $\vec{r}.(2\hat{i}+6\hat{j})+12=0$ and $\vec{r}.(3\hat{i}-\hat{j}+4\hat{k})=0$ which are at unit distance from the origin.
11. Find the foot of the perpendicular from (1, 6, 3) on the line $\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}$.
12. Find the Cartesian and the vector equations of the planes through the intersection of the planes $\vec{r}.(\hat{i}-\hat{j})+6=0$ and $\vec{r}.(3\hat{i}+3\hat{j}-4\hat{k})=0$ which are at unit distance from the origin.
13. Find the shortest distance and the equation of the line of shortest distance between the following two lines: $\vec{r}=(-\hat{i}+\hat{j}+9\hat{k})+\lambda(2\hat{i}+\hat{j}-3\hat{k})$ and $\vec{r}=(3\hat{i}-15\hat{j}+9\hat{k})+\mu(2\hat{i}-7\hat{j}+5\hat{k})$.

14. Find the vector equation of the plane passing through the intersection of the planes $\vec{r}.(2\hat{i}-7\hat{j}+4\hat{k})=3$ and $\vec{r}.(3\hat{i}-5\hat{j}+4\hat{k})+11=0$ and passing through the point (-2, 1, 3).
15. Find the image of the point (1, 3, 4) in the plane x - y + z = 5.
16. Find the vector equation of a line passing through the point with position vector $(2\hat{i}-3\hat{j}-5\hat{k})$ and perpendicular to the plane $\vec{r}.(6\hat{i}-3\hat{j}+5\hat{k})+2=0$. Also, find the point of intersection of this line and the plane.
17. Find the distance of the point (2, 3, 4) from the plane $3x+2y+2z+5=0$, measured parallel to the line $\frac{x+3}{3}=\frac{y-2}{6}=\frac{z}{2}$.
18. Find the image of the point (1, 6,3) in the line $\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}$.
19. Show that the line L, whose vector equation is $\vec{r}=(2\hat{i}-2\hat{j}+3\hat{k})+\lambda(\hat{i}-\hat{j}+4\hat{k})$, is parallel to the plane π whose vector equation is $\vec{r}.(\hat{i}+5\hat{j}+\hat{k})=5$, and find the distance between them.
20. The position vectors of two points A and B are $3\hat{i}+\hat{j}+2\hat{k}$ and $\hat{i}-2\hat{j}-4\hat{k}$ respectively. Find the vector equation of the plane passing through B and perpendicular to the vector AB.
21. Find the shortest distance between the following lines :
$\vec{r}=(1+t)\hat{i}+(t-2)\hat{j}+(3-2t)\hat{k}$ and $\vec{r}=(s-1)\hat{i}+(2s-1)\hat{j}-(2s+1)\hat{k}$.
22. Show that the lines $\frac{x-5}{4}=\frac{y-7}{4}=\frac{z+3}{-5}$ and $\frac{x-8}{7}=\frac{y-4}{1}=\frac{z-5}{3}$ intersect each other.
23. Show that the lines $\frac{x-3}{3}=\frac{y+1}{2}=\frac{z-1}{5}$ and $\frac{x-2}{4}=\frac{y-1}{3}=\frac{z+1}{-2}$ do not intersect each other.
24. Find the image of the point (3, 5,3) in the line $\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}$.
25. Find the length and the foot of the perpendicular from the point (1, 1, 2) to the plane $\vec{r}.(2\hat{i}-2\hat{j}+4\hat{k})+5=0$.
26. Find the vector equation of the line passing through the point A (2, -1, 1) and parallel to the joining the points B (-1, 4, 1) and C (1, 2, 2). Also find the Cartesian equation of the line.
27. Find the equation of the plane passing through the point (1, 1,1) and perpendicular to each of the following planes : $x+2y+3z=7$ and $2x-3y+4z=0$.
28. The Cartesian equations of the line are $6x-2=3y+1=2z-2$.
Find (a) the direction ratios of the line, and (b) Cartesian and vector equations of the line parallel to this line and passing through the point (2, -1, -1).
29. Find the foot of the perpendicular from the point (0, 2, 3) on the line $\frac{x+3}{5}=\frac{y-1}{2}=\frac{z+4}{3}$. Also find the length of the perpendicular.

30. Find the equation of the plane passing through the line of intersection of the planes $2x+y-z=3, 5x-3y+4z+9=0$ and parallel to the line $\frac{x-1}{2}=\frac{y-3}{4}=\frac{z-5}{5}$.
31. Find the Cartesian and vector equations of the line which passes through the point (1, 2, 3) and is parallel to the line $\frac{-x-2}{1}=\frac{y+3}{7}=\frac{2z-6}{3}$.
32. Show that the lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and $\frac{x-4}{5}=\frac{y-1}{2}=z$ intersect. Find the point of intersection also.
33. The Cartesian equation of a line are $3x+1=6y-2=1-z$. Find the fixed point through which it passes, it direction ratios and also its vector equation.
34. Find the equation of the plane passing through the points (0, -1, 0), (1, 1, 1) and (3,3,0).
35. Find the equation of the plane passing through the point (-1, -1, 2) and perpendicular to the planes $3x+2y-3z=1$ and $5x-4y+z=5$.
36. Find the equation of the line passing through the point (2, 1, 3) and perpendicular to the line $\frac{x-1}{1}=\frac{y-2}{2}=\frac{z-3}{3}$ and $\frac{x}{-3}=\frac{y}{2}=\frac{z}{5}$.
37. Find the length of the perpendicular drawn from the point (2, 3, 7) to the plane $3x-y-z=7$. Also find the coordinates of the foot of the perpendicular.
38. Find the equation of the line through the point (-1, 2, 3) which is perpendicular to the line $\frac{x}{2}=\frac{y-1}{-3}=\frac{z+2}{-2}$ and $\frac{x+3}{-1}=\frac{y+2}{2}=\frac{z-1}{3}$.
39. Prove that the equation of the plane making intercepts a, b and c on the co-ordinates axes, is of the form $\frac{x}{a}=\frac{y}{b}=\frac{z}{c}=1$.
40. Find the co-ordinates of the foot of the perpendicular drawn from the point A (1, 8, 4) to the line joining the points B (0, -1, 3) and C (2, -3, -1).
41. Find the perpendicular distance of the point (1, 0, 0) from the line $\frac{x-1}{2}=\frac{y+1}{-3}=\frac{z+10}{8}$.
42. Find the equation of the line passing through the point P(-1, 3, -2) and perpendicular to the lines $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ and $\frac{x+2}{-3}=\frac{y-1}{2}=\frac{z+1}{5}$.
43. Find the equation of the plane passing through the points P(1, -1, 2) and Q(2, -2, 2) and perpendicular to the plane $6x-2y+2z=9$.
44. Find the vector and Cartesian equations of the plane passing through the point (1, 2, 3) and perpendicular to the line with direction ratios 2, 3, -4.
45. Find the length of the perpendicular drawn from the point (1, 2, 3) on the line $\frac{x-6}{3}=\frac{y-7}{2}=\frac{z-7}{-2}$.
46. Find the foot of the perpendicular drawn from the point P(1, 6, 3) on the line $\frac{x}{1}=\frac{y-1}{2}=\frac{z-3}{3}$. Also find its distance from point P.

47. A plane passes through a fixed point A(a, b, c). Show that the locus of the foot of the perpendicular from the origin on the plane is $x^2+y^2+z^2-ax-by-cz=0$.
48. Find the equation of the plane passing through the points (1, 2, 3) and (0, -1, 0) and parallel to the line $\frac{x-1}{2}=\frac{y+2}{3}=\frac{z}{-3}$.
49. The vector equations of two lines are $\vec{r}=\hat{i}+2\hat{j}+3\hat{k}+\lambda(\hat{i}-3\hat{j}+2\hat{k})$ and $\vec{r}=4\hat{i}+5\hat{j}+6\hat{k}+\mu(2\hat{i}+3\hat{j}+\hat{k})$. Find the shortest distance between the above lines.
50. Find the equation of the plane passing through the points (0, -1, -1), (4, 5, 1) and (3, 9, 4).
51. Find the coordinates of the point where the line $\frac{x+1}{2}=\frac{y+2}{3}=\frac{z+3}{4}$ meets the plane $x+y+4z=6$.
52. Find the image of the point (1, 2, 3) in the plane $x+2y+4z=38$.
53. Find the equation of the plane passing through the points (1, -1, 2) and (2, -2, 2) and is perpendicular to the plane $6x-2y+2z=9$.
54. Find the equation of the plane passing through the points (3, 4, 2), (2, -2, -1) and (7, 0, 6).
55. Find the equation of the plane primary through the intersection of the plane $\vec{r}.(2\hat{i}+\hat{j}+3\hat{k})=7$, $\vec{r}.(2\hat{i}+5\hat{j}+3\hat{k})=9$ and the point (2, 1, 3).
56. Find the equation of the plane which is perpendicular to the plane $5x+3y+6z+8=0$ and which contains the line of intersection of the planes $x+2y+3z-4=0$ and $2x+y-z+5=0$.
57. Find the equation of the line, which is parallel to $2\hat{i}-\hat{j}+3\hat{k}$ and which passes through the point (5, -2, 4).
58. Find the vector equation of the line passing through the point (1, 2, -4) and perpendicular to the two lines : $\frac{x-8}{3}=\frac{y+19}{-16}=\frac{z-10}{7}$ and $\frac{x-15}{3}=\frac{y-29}{8}=\frac{z-5}{-5}$.
59. Find the distance of the point with position vector from the point of intersection of the line $\vec{r}=(2\hat{i}-\hat{j}+2\hat{k})+\lambda(3\hat{i}+4\hat{j}+12\hat{k})$ and the plane $\vec{r}.(\hat{i}-\hat{j}+\hat{k})=5$.
60. The Cartesian equation of a line AB is $\frac{2x-1}{\sqrt{3}}=\frac{y+2}{2}=\frac{z-3}{3}$. Find the direction cosines of a line parallel to AB.
61. Find the shortest distance between the lines : $\vec{r}=(\hat{i}+2\hat{j}+3\hat{k})+\lambda(2\hat{i}+3\hat{j}+4\hat{k})$ and $\vec{r}=2\hat{i}+4\hat{j}+5\hat{k}+\mu(3\hat{i}+4\hat{j}+5\hat{k})$.
62. Find the distance of the point (1, -2, 3) from the plane $x-y+z=5$ measured parallel to the line $\frac{x}{2}=\frac{y}{3}=\frac{z}{-6}$.
63. Find the shortest distance between the following lines: $\vec{r}=(\hat{i}+2\hat{j}+\hat{k})+\lambda(\hat{i}-\hat{j}+\hat{k})$ and $\vec{r}=2\hat{i}-\hat{j}+\hat{k}+\mu(2\hat{i}+\hat{j}+2\hat{k})$.
64. If the equation of the line AB is $\frac{x-3}{1}=\frac{y+2}{-2}=\frac{z-5}{4}$, find the direction ratios of a line parallel to AB.

65. Find the equation of the line passing through the point P(4, 6, 2) and the point of intersection of the line $\frac{x-1}{3}=\frac{y}{2}=\frac{z+1}{7}$ and the plane $x+y-z=8$.

66. Find the distance of the point (-2, 3, -4) from the lines $\frac{x+2}{3}=\frac{2y+3}{4}=\frac{3z+4}{5}$ measured parallel to the plane $4x+12y-3z+1=0$.

67. Find the shortest distance between the following lines: $\frac{x-3}{1}=\frac{y-5}{-2}=\frac{z-7}{1}$ and $\frac{x+1}{7}=\frac{y+1}{-6}=\frac{z+1}{1}$.

68. Find the point on the line $\frac{x+2}{3}=\frac{y+1}{2}=\frac{z-3}{2}$ at a distance $3\sqrt{2}$ from the point (1, 2,3)

69. Find the equation of the plane passing through the point (-1, -1, 2) and perpendicular to each of the following planes: $2x+3y-3z=2$ and $5x-4y+z=6$.

70. Find the equation of the plane passing through the points (3, 4, 1) and (0, 1, 0) are parallel to the line $\frac{x+3}{2}=\frac{y-3}{7}=\frac{z-2}{5}$.

Note : if any mistake on this, kindly inform on the mail id : bknal1207@gmail.com

Your Observation! Our Correction !!