IIT-JEE / ENGINEERING

MATHS

Assignment 1.0

Matrices

By

BHARAT BHUSHAN @ B. K. NAL
Assistant Professor (Computer Science)
Director, BSTI, Kokar

&

SUPRIYA BHARATI
Assistant Professor (Computer Science)
Asst. Director, BSTI, Kokar

Buddha Science & Technical Institute
Kokar, Ranchi-834001, Jharkhand, India
www.bharatsir.com

Assignment 1.0

1. Express the matrix $A=\begin{bmatrix}3 & -4\\ 1 & -1\end{bmatrix}$ as the sum of a symmetric and a skew symmetric matrix. [C.B.S.E. 1998]

2. Express the matrix $A=\begin{bmatrix}3 & 1\\ -4 & -1\end{bmatrix}$ as the sum of a symmetric and a skew symmetric matrix. [C.B.S.E. 1998]

3. For the matrix $A=\begin{bmatrix}-3 & 6 & 0\\ 4 & -5 & 8\\ 0 & -7 & -2\end{bmatrix}$ Find $\frac{1}{2}(A-A')$, where A' is the transpose of matrix A. [A.I.C.B.S.E. (Comptt.)1998]

4. For the matrix $A=\begin{bmatrix}-2 & 3 & 4\\ 5 & -4 & -3\\ 7 & 2 & 9\end{bmatrix}$ Find $\frac{1}{2}(A-A')$, where A' is the transpose of matrix A. [A.I.C.B.S.E. (Comptt.)1998]

5. If $A=\begin{bmatrix}1 & -3 & 2\\ 2 & 0 & 2\end{bmatrix}$, $B=\begin{bmatrix}2 & -1 & -1\\ 1 & 0 & -1\end{bmatrix}$, Find the matrix C such that A+B+C is a zero matrix. [C.B.S.E. 1999]

6. Construct a 2 X 3 matrix whose elements in the *i*th row and the *j*th column are given by $a_{ij}=\frac{3i-j}{2}$. [C.B.S.E. 1999]

7. If $A=\begin{bmatrix}2 & 2\\ -3 & 1\\ 4 & 0\end{bmatrix}$, $B=\begin{bmatrix}6 & 2\\ 1 & 3\\ 0 & 4\end{bmatrix}$, Find the matrix C such that A+B+C is a zero matrix. [C.B.S.E. 1999]

8. Construct a 3 X 2 matrix whose elements in the *i*th row and the *j*th column are given by $a_{ij}=\frac{3i+j}{2}$. [C.B.S.E. 1999]

9. If $A=\begin{bmatrix}2 & 0 & 2\\ 1 & 0 & -1\end{bmatrix}$, $B=\begin{bmatrix}-3 & 1 & 0\\ 2 & 0 & 1\end{bmatrix}$, Find the matrix C such that A+B+C is a zero matrix. [A.I.C.B.S.E. 1999]

10. Construct a 2 X 3 matrix whose elements in the *i*th row and the *j*th column are given by $a_{ij}=\frac{2i+3j}{2}$. [A.I.C.B.S.E. 1999]

11. If $A=\begin{bmatrix}1 & 3\\ 2 & 1\\ 3 & -1\end{bmatrix}$, $B=\begin{bmatrix}2 & 1\\ 1 & 2\\ -1 & 0\end{bmatrix}$, Find the matrix C such that A+B+C is a zero matrix. [A.I.C.B.S.E. 1999]

12. Construct a 2 X 3 matrix whose elements in the *i*th row and the *j*th column are given by $a_{ij}=\frac{i+3j}{2}$. [A.I.C.B.S.E. 1999]

13. If $f(x)=x^2-4x+1$, find $f(A)$ when $A=\begin{bmatrix}2 & 3\\ 1 & 2\end{bmatrix}$. [A.I.C.B.S.E. 1999]

14. If $f(x)=x^2-2x-3$, find $f(A)$ when $A=\begin{bmatrix}1 & 2\\ 2 & 1\end{bmatrix}$. [A.I.C.B.S.E. 2005]

15. If $f(x)=x^2-5x+7$, find $f(A)$ when $A=\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}$. [A.I.C.B.S.E. 2003]

16. Find a matrix X such that 2A + B + X = 0, where $A=\begin{bmatrix}-1 & 2\\ 3 & 4\end{bmatrix}$, $B=\begin{bmatrix}3 & -2\\ 1 & 5\end{bmatrix}$. [C.B.S.E. 2000, 2002]

17. Find a matrix X such that A + 2B + X = 0, where $A=\begin{bmatrix}2 & -1\\ 3 & 5\end{bmatrix}$, $B=\begin{bmatrix}-1 & 1\\ 0 & 2\end{bmatrix}$. [C.B.S.E. 2000]

18. Find a matrix X such that 2A - B + X = 0, where $A=\begin{bmatrix}3 & 1\\ 0 & 2\end{bmatrix}$, $B=\begin{bmatrix}-2 & 1\\ 0 & 3\end{bmatrix}$. [A.I.C.B.S.E. 2000]

19. Find a matrix X such that 3A - 2B + X = 0, where $A=\begin{bmatrix}4 & 2\\ 1 & 3\end{bmatrix}$, $B=\begin{bmatrix}-2 & 1\\ 3 & 2\end{bmatrix}$. [A.I.C.B.S.E. 2000]

20. If $A=\begin{bmatrix}i & 0\\ 0 & -i\end{bmatrix}$ and $B=\begin{bmatrix}0 & i\\ i & 0\end{bmatrix}$, show that $AB\neq BA$.

21. If $A=\begin{bmatrix}2 & 3\\ 4 & 5\end{bmatrix}$, prove that A-A$'$ is a skew symmetric matrix where A^T denotes the transpose of A. [A.I.C.B.S.E. 2000]

22. If $A=\begin{bmatrix}1 & 4\\ 3 & 7\end{bmatrix}$, show that A-A$'$ is a skew symmetric matrix where A$'$ is the transpose of matrix A. [C.B.S.E. 2001]

23. If $A=\begin{bmatrix}4 & 1\\ 5 & 8\end{bmatrix}$, show that A + A$'$ is a skew symmetric matrix where A$'$ is the transpose of matrix A. [C.B.S.E. 2001]

24. If $A=\begin{bmatrix}3 & 4\\ 5 & 1\end{bmatrix}$, show that A - A$'$ is a skew symmetric matrix where A$'$ is the transpose of matrix A. [A.I.C.B.S.E. 2001]

25. If $A=\begin{bmatrix}3 & -4\\ 1 & -1\end{bmatrix}$, show that (A - A$'$) is a skew symmetric matrix where A$'$ is the transpose of matrix A. [A.I.C.B.S.E. 2001]

26. If $A=\begin{bmatrix}2 & 4\\ 5 & 6\end{bmatrix}$, show that (A - A$'$) is a skew symmetric matrix where A$'$ is the transpose of matrix A. [A.I.C.B.S.E. 2001]

27. If $A=\begin{bmatrix}1 & 3\\ 2 & 4\end{bmatrix}$, show that (A - A$'$) is a skew symmetric matrix where A$'$ is the transpose of matrix A. [A.I.C.B.S.E. (Comptt.)2001]

28. If $A=\begin{bmatrix}-1\\ 2\\ 3\end{bmatrix}$ and $B=\begin{bmatrix}-2 & -1 & -4\end{bmatrix}$, verify that $(AB)'=B'A'$. [C.B.S.E. 2002]

29. If $A=\begin{bmatrix}3\\ 5\\ 2\end{bmatrix}$ and $B=\begin{bmatrix}1 & 0 & 4\end{bmatrix}$, verify that $(AB)'=B'A'$. [C.B.S.E. 2002]

30. Construct a 2 X 3 matrix A, whose elements are given by $a_{ij}=\frac{(i-2j)^2}{2}$.

31. Construct a 3 X 2 matrix A, whose elements are given by $a_{ij}=\frac{(i-2j)^2}{2}$.

32. From the following equations, find the values of x and y :

$$2\begin{bmatrix}x & 5\\ 7 & y-3\end{bmatrix}+\begin{bmatrix}3 & 4\\ 1 & 2\end{bmatrix}=\begin{bmatrix}7 & 14\\ 15 & 14\end{bmatrix}$$ [C.B.S.E. (Comptt.)2002]

33. If $A=\begin{bmatrix}3 & -2\\ 4 & -2\end{bmatrix}$, find K such that $A^2=KA-2I_2$. [C.B.S.E. 2003]

34. Solve for x and y, given that $\begin{bmatrix}2 & -3\\ 1 & 1\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}1\\ 3\end{bmatrix}$. [C.B.S.E.(Comptt.) 2003]

35. Solve for x and y, given that $\begin{bmatrix} x & y \\ 3y & x \end{bmatrix}\begin{bmatrix} 1 \\ 2 \end{bmatrix}=\begin{bmatrix} 3 \\ 5 \end{bmatrix}$. [C.B.S.E.(Comptt.) 2003]

36. Find the values of a and b for which the following holds $\begin{bmatrix} 4 & 2 \\ 3 & -1 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix}=\begin{bmatrix} -4 \\ 2 \end{bmatrix}$.

37. If $A=\begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$, show that $A^2-5A-14I=0$. [C.B.S.E. 2004]

38. If $X=\begin{bmatrix} \cos A & \sin A \\ -\sin A & \cos A \end{bmatrix}$, then prove that $X^n=\begin{bmatrix} \cos nA & \sin nA \\ -\sin nA & \cos nA \end{bmatrix}; n\in N$.

39. If $A=\begin{bmatrix} 3 & 4 \\ -4 & -3 \end{bmatrix}$, find $f(A)$; where $f(x)=x^2-5x+7$. [C.B.S.E.(Comptt.) 2004]

40. If $A=\begin{bmatrix} -1 & 2 \\ 3 & 1 \end{bmatrix}$, find $f(A)$; where $f(x)=x^2-2x+3$. [C.B.S.E.(Comptt.) 2004]

41. If $A=\begin{bmatrix} -4 & 1 \\ 3 & 2 \end{bmatrix}$, find $f(A)$; where $f(x)=x^2-2x+3$. [C.B.S.E.(Comptt.) 2004]

42. If $A=\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$, prove that $A^3-4A^2+A=0$. [C.B.S.E. 2005]

43. If $A=\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$, $B=\begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix}$, find $3A^2-2B+I$. [C.B.S.E. 2005]

44. If $A=\begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}$, find K such that $A^2-8A+KI=0$.

45. If $A=\begin{bmatrix} \cos\theta & i\sin\theta \\ i\sin\theta & \cos\theta \end{bmatrix}$, then prove by principle of Mathematical Induction that $A^n=\begin{bmatrix} \cos n\theta & i\sin n\theta \\ \sin n\theta & \cos n\theta \end{bmatrix}$; *where* $n\in N$. [A.I.C.B.S.E. 2005]

46. Find the value of x, if $\begin{bmatrix} 1 & x & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 3 & 2 & 5 \end{bmatrix}\begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}=0$. [A.I.C.B.S.E. 2005]

47. If $A=\begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}$, find K so that $A^2=8A+KI$. [C.B.S.E.(Comptt.) 2005]

48. Solve for x and y : $\begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} 3 \\ 11 \end{bmatrix}$. [A.I.C.B.S.E. 2005]

49. Find the value of x if $\begin{bmatrix} 1 & x & 1 \end{bmatrix}\begin{bmatrix} 1 & 3 & 2 \\ 2 & 5 & 1 \\ 15 & 3 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ x \end{bmatrix}=0$.

[C.B.S.E.(Comptt.) 2006]

50. Express the following matrix as sum of a symmetric and a skew symmetric matrix :

$$\begin{bmatrix} 1 & 3 & 5 \\ -6 & 8 & 3 \\ -4 & 6 & 5 \end{bmatrix}.$$ [C.B.S.E.2006; A.I.C.B.S.E.2006]

51. If $A = \begin{bmatrix} 3 & -4 \\ -1 & 2 \end{bmatrix}$, find a matrix B such that AB = I. [C.B.S.E.2006]

52. If $A = \begin{bmatrix} 1 & 2 \\ 4 & 1 \end{bmatrix}$, find the value of $A^2 + 2A + 7I$. [C.B.S.E.(Foreign)2006]

53. Show that the matrix $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ satisfies the equation $A^2 - 5A + 7I = 0$.

54. If $A = \begin{bmatrix} 2 & -3 \\ 3 & 4 \end{bmatrix}$, show that $A^2 - 6A + 17I = 0$. [C.B.S.E.2007]

55. Find the value of x and y if : $2\begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$

56. Let $A = \begin{bmatrix} 3 & 2 & 5 \\ 4 & 1 & 3 \\ 0 & 6 & 7 \end{bmatrix}$. Express A as sum of two matrices such that one is symmetric and the other is skew symmetric. [C.B.S.E.(Delhi)2008]

57. If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, verify that $A^2 - 4A - 5I = 0$. [C.B.S.E.(Delhi)2008]

58. If $\begin{bmatrix} x+3y & y \\ 7-x & 4 \end{bmatrix} = \begin{bmatrix} 4 & -1 \\ 0 & 4 \end{bmatrix}$, find the values of x and y. [A.I.C.B.S.E.2008]

59. Using elementary transformations, find the inverse of the following matrix :

$$\begin{bmatrix} 2 & -1 & 4 \\ 4 & 0 & 2 \\ 3 & -2 & 7 \end{bmatrix}.$$ [A.I.C.B.S.E.2008]

60. Using elementary transformations, find the inverse of the following matrix :

$$\begin{bmatrix} 2 & 5 & 3 \\ 3 & 4 & 1 \\ 1 & 6 & 2 \end{bmatrix}.$$ [A.I.C.B.S.E.2008]

Note : if any mistake on this kindly inform on the mail id : bknal1207@gmail.com