IIT-JEE / ENGINEERING

MATHS

Assignment 1.0

Continuity and Differentiability

By

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Assignment 1.0

1. Verify Rolle's theorem for the function $f(x)=x^2-x-6$ in the interval [-2,3].
2. Differentiate $\tan^{-1}\left[\frac{\cos x}{1+\sin x}\right]$ w.r.t. x.
3. Verify Rolle's theorem for the function $f(x)=x^2-4x+3$ in the interval [1,3].
4. Verify Rolle's theorem for the function $f(x)=x^2-6x+5$ in the interval [1,5].
5. Differentiate $\tan^{-1}\left[\frac{1-\cos x}{\sin x}\right]$ w.r.t. x.
6. Verify Rolle's theorem for the function $f(x)=x^2-2x-3$ in the interval [-1,3].
7. Verify Rolle's theorem for the function $f(x)=x^3-7x^2+16x-12$ in the interval [2,3].
8. Verify Rolle's theorem for the function $f(x)=x^3+3x^2-24x-80$ in the interval [-4,5].
9. If $y=\tan^{-1}x$, show that $(1+x^2)\frac{d^2y}{dx^2}+2x\frac{dy}{dx}=0$.
10. Discuss the applicability of Rolle's theorem for the function $f(x)=x^{2/3}$ on interval [-1,1]
11. If $y=\cot x$, show that $\frac{d^2y}{dx^2}+2x\frac{dy}{dx}=0$.
12. If $y=\operatorname{cosec}x+\cot x$, show that $\sin x\frac{d^2y}{dx^2}=y^2$.
13. If $y=\sec x-\tan x.$, show that $\cos x\frac{d^2y}{dx^2}=y^2$.
14. Differentiate $\cos^{-1}\theta$ with respect to $\log(1+\theta)$.
15. If $f(x)=\begin{cases}\frac{|x|}{x}, & x\neq 0 \text{ and } 0, \text{ for x=0 find whether f(x)is continuous at x=0.}\end{cases}$
16. If $x=a(\theta-\sin\theta)\ and\ y=a(1-\cos\theta).$ find $\frac{d^2y}{dx^2}\ at\ \theta=\frac{\pi}{2}$
17. Differentiate $\log(1+\theta)$ with respect to $\sin^{-1}\theta$.
18. Differentiate $\sin^{-1}\theta$ with respect to $\log(1+\theta)$.
19. **If** $x^py^q=(x+y)^{p+q}$, prove that $\frac{dy}{dx}=\frac{y}{x}$
20. Find if Lagrange's mean value theorem if applicable to the function $f(x)=x+\frac{1}{x}\ on[1,3]$.

21. Using Rolle's theorem , find the point on the curve $y = x^2$, $x \in [-2,2]$where tangent is parallel to x-axis.

22. Find when $\frac{d^2y}{dx^2}$ when $y = -\log\left(\frac{x^2}{e}\right)$

23. Using Rolle's theorem , find the point on the curve $y = 16 - x^2$, $x \in [-1,1]$where tangent is parallel to x-axis.

24. Find $\frac{dy}{dx}, when\ x^y = e^{x-y}$

25. For what value of K, is the following function continuous at x=0

$$f(x) = \begin{cases} \frac{1-\cos 4x}{8x^2} \ for\ x \neq 0 \ and\ K \ for\ x = 0 \end{cases}$$

26. If $y = \frac{5x}{\sqrt[3]{1-x^2}} + \sin^2(2x+3)$, *find* $\frac{dy}{dx}$.

27. Differentiate e^{x^2} w.r.t x.

28. Differentiate $e^{\sin^{-1}x}$ w.r.t x.

29. Verify Rolle's theorem for the function $f(x) = x^2 - x - 12$ in the interval [-3,4].

30. Show that $f(x) = 2x - |x|$ is continuous at x=0.

31. If $y = e^x(\sin x + \cos x)$, prove that $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$

32. If the function $f(x) = \{3ax + b \ if\ x > 1,\ 11 \ if\ x = 1, and\ 5ax - 2b \ if\ x < 1\}$ is continuous at x=1,find the value of a and b.

33. Discuss the continuity of function $f(x)$ at $x=0$ if $f(x)$ $f(x) = \{2x - 1 \ if\ x < 0,\ 2x + 1 \ if\ x \geq 0.$

34. Verify the Lagrange's mean value theorem for function $f(x) = \sqrt{x^4 - 4}$ in the interval [2,4]

35. If $y = \sqrt{\frac{1-\sin 2x}{1+\sin 2x}}$ show that $\frac{dy}{dx} + \sec^2\left(\frac{\pi}{4} - x\right) = 0$.

36. Verify the Lagrange's mean value theorem for function $f(x) = x^2 + x - 1$ in the interval [0,4]

37. If $y = \log\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)$, show that

38. Verify Rolle's theorem for the function $f(x) = x^2 - 5x + 6$ in the interval [2,3].

39. Differentiate the following with respect to x: $\log\left(x+\sqrt{1+x^2}\right)$.

40. Differentiate $\log\sin\sqrt{1+x^2}$ with respect to x.

41. If $y=\log\sqrt{\frac{1-\cos x}{1+\cos x}}$, show that $\frac{dy}{dx}=\cos ecx$.

42. Differentiate $e^{\sin x}+(\tan x)^x$ with respect to x.

43. Differentiate $\tan^{-1}\left[\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right]$ with respect to x.

44. If $y=\log\left(\sqrt{\frac{1+\cos 2x}{1-e^{2x}}}\right)$ find $\frac{dy}{dx}$.

45. If $x=a(\theta-\sin\theta)$ *and* $y=a(1-\cos\theta)$. find $\frac{dy}{dx}$ $at\ \theta=\frac{\pi}{2}$

46. If $y=\tan^{-1}\left(\sqrt{\frac{1+\sin x}{1-\sin x}}\right)$, find $\frac{dy}{dx}$.

47. $y=\log\left(\sqrt{\frac{1+\sin^2 x}{1-\tan x}}\right)$, find $\frac{dy}{dx}$.

48. If $x=a(\cos\theta+\theta\sin\theta)$, $y=a(\sin\theta-\theta\cos\theta)$ find $\frac{dy}{dx}$.

49. Differentiate $\cot^{-1}\left[\frac{1-x}{1+x}\right]$ with respect to x.

50. prove that $\frac{d}{dx}\left[\frac{x}{2}\sqrt{a^2-x^2}+\frac{a}{2}\sin^{-1}\frac{x}{a}\right]=\sqrt{a^2-x^2}$.

51. Verify the Lagrange's mean value theorem for function $f(x)=x^2-2x+4$ in the interval [1,5].

52. Differentiate $\tan^{-1}\left[\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right]$ with respect to x.

53. Differentiate $\tan^{-1}\left[\frac{\sqrt{1+x^2}-1}{x}\right]$ with respect to x.

54. Find $\frac{dy}{dx}$ *if* $y=(x)^{\cos x}+(\sin x)^{\tan x}$.

55. Find $\frac{dy}{dx}$ *if* $y=\tan^{-1}\sqrt{\frac{1+\sin x}{1-\sin x}}$.

56. Find $\frac{dy}{dx}$ *if* $y=\sin^{-1}\left(\frac{5x+12\sqrt{1-x^2}}{13}\right)$.

57. *If* $y=(\sin x)^x+(\cos x)^{\tan x}$, find $\frac{dy}{dx}$.

58. Differentiate $\tan^{-1}\frac{2x}{1-x^2}$ with respect to $\sin^{-1}\frac{2x}{1+x^2}$.

59. *If* $y=(\log x)^x+x^{\log x}$, find $\frac{dy}{dx}$.

60. If $x=a\left(\frac{1+t^2}{1-t^2}\right)$ *and* $y=\frac{2t}{1-t^2}$, find $\frac{dy}{dx}$.

61. If $y=\left(x+\sqrt{x^2+a^2}\right)^n$, prove that $\frac{dy}{dx}=\frac{ny}{\sqrt{x^2+a^2}}$.

62. If $f(x)=\left(\frac{3+x}{1+x}\right)^{2+3x}$, find $f'(0)$.

63. If $x=3\sin t-\sin 3t, y=3\cos t-\cos 3t$. find $\frac{d^2y}{dx^2}$ *at* $t=\frac{\pi}{3}$.

64. Differentiate $x^{-x}\sin^{-1}\sqrt{x}$ with respect to x.

65. If $x=2\cos\theta-\cos 2\theta, y=2\sin\theta-\sin 2\theta$, find $\frac{d^2y}{dx^2}$ *at* $\theta=\frac{\pi}{2}$.

66. If $y=x^{\cos x}+(\cos x)^{\sin x}$ find $\frac{dy}{dx}$.

67. If $y=x^x$ show that $\frac{d^2y}{dx^2}-\frac{1}{y}\left(\frac{dy}{dx}\right)^2-\frac{y}{x}=0$.

68. Differentiate $x^{\tan x}+\sqrt{\frac{x^2+1}{x}}$ with respect to x.

69. Differentiate $(\sin x)^{\tan x}+(\cos x)^{\sec x}$ w. r. t. x.

70. If $y=\sqrt{\frac{(x-3)(x^4+4)}{3x^2+4x+5}}$ find $\frac{dy}{dx}$.

71. If $x=a\sin 2t(2+\cos 2t), y=b\cos 2t(1-\cos 2t)$. find $\left(\frac{dy}{dx}\right)_{at\,t=\frac{\pi}{4}}=\frac{b}{a}$

72. If $y\sqrt{x^2+1}=\log\left[\sqrt{x^2+1}-x\right]$ show that $(x^2+1)\frac{dy}{dx}+xy+1=0$.

73. If $y=\sqrt{x}+\frac{1}{\sqrt{x}}$ show that $2x\frac{dy}{dx}+y=2\sqrt{x}$.

74. If $y=\tan^{-1}\left[\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right]$, find $\left(\frac{dy}{dx}\right)$.

75. Verify Rolle's theorem for the function $f(x)=(x-1)(x-2)^2$ in the interval $[1,2]$.

76. Verify Rolle's theorem for the function $f(x)=\sin x+\cos x$ in the interval $\left[0,\frac{\pi}{2}\right]$.

77. Verify Lagrane's theorem for the function $f(x)=x^3+2x+3,[4,6]$.

78. If $y = a\cos(\log x) + b\sin(\log x)$, show that $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$.

79. If $y = \sqrt{\frac{\tan^{-1} x.(x^2+1)}{\sin x^3}}$, find $\left(\frac{dy}{dx}\right)$.

80. If $y = \sin(\log x)$, prove that $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$.

81. Verify Rolle's theorem for the function $f(x) = x^2 - 5x + 4$ in the interval $[1,4]$.

82. If $y = 3e^{2x} + 2e^{3x}$, prove that $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0$.

83. If $f(x) = \left\{\frac{x^2-25}{x-5}\right.$ when $x \neq 5$, K when $x = 5$ is continuous at x=5, find the value of K.

84. Let $f(x) = \left\{\frac{1-\sin^2 x}{3\cos^2 x}\right.$ *if* $x < \frac{\pi}{2}$, a *if* $x = \frac{\pi}{2}$, $\frac{b(1-\sin x)}{(\pi - 2x)^2}$ *if* $x > \frac{\pi}{2}$, if f(x) is continuous at $x = \frac{\pi}{2}$, find a and b.

85. If $y = \left[\log\left(x + \sqrt{1+x^2}\right)\right]^2$, show that $(1+x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - 2 = 0$.

86. If $y = x^x + (\sin x)^x$, find $\left(\frac{dy}{dx}\right)$.

87. Discuss the continuity of following function at x=0.

$$f(x) = \left\{\frac{x^4 + 2x^3 + x^2}{\tan^{-1} x}\right. \text{ for } x \neq 0, 0 \text{ for } x = 0.$$

88. Verify Lagrane's mean value theorem for the following function:
$f(x) = x^2 + 2x + 3$, for interval [4,6].

89. If $f(x) = \sqrt{\frac{\sec x - 1}{\sec x + 1}}$ find $f'(x)$. Also find $f'\left(\frac{\pi}{2}\right)$.

90. If $x\sqrt{1+y} + y\sqrt{1+x} = 0$, find $\frac{dy}{dx}$.

91. For what value of K is the following function is continuous at x = 2 ?

$$f(x) = \{2x+1; x < 2,\ K; x = 2,\ 3x-1; x > 2$$

92. Show that the rectangle of maximum area that can be inscribed in a circle is a square.

93. Show that height of the cylinder of maximum volume that can be inscribed in a cone of height h is $\frac{1}{3}h$.

94. If $y = \sqrt{x^2+1} - \log\left(\frac{1}{x} + \sqrt{1+\frac{1}{x}}\right)$ find $\frac{dy}{dx}$.

95. If $y = \cot^{-1}\left[\dfrac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right]$ find $\dfrac{dy}{dx}$.

Note : if any mistake on this, kindly inform on the mail id : bknal1207@gmail.com

Your Observation! Our Correction !!

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