IIT-JEE / ENGINEERING

MATHS

Assignment 1.0

Applications of Derivatives

By

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Assignment 1.0

1. For the function $f(x) = -2x^3 - 9x^2 - 12x + 1$,find the interval (S):
(A) In which $f(x)$ is increasing. (b) In which $f(x)$ is decreasing.

2. A rectangle is inscribed in a semicircle with one of its sides on the diameter of semicircle. Find the dimension of the rectangle so that its area is maximum. Find also the area.

3. For the function $f(x) = x^3 - 6x^2 - 1$,find the interval (S):
(a) In which $f(x)$ is increasing. (b) In which $f(x)$ is decreasing.

4. For the function $f(x) = 2x^3 - 8x^2 + 10x + 5$,find the interval (S):
(a) In which $f(x)$ is increasing. (b) In which $f(x)$ is decreasing.

5. Show that the surface area of closed cuboid with square baseand given value is minimum, when it is cube.

6. For the function $f(x) = 2x^3 - 12x^2 + 36x + 17$,find the interval (S):
(a) In which $f(x)$ is increasing. (b) In which $f(x)$ is decreasing

7. For the function $f(x) = x^3 - 6x^2 - 36x + 24$,find the interval (S):
(a) In which $f(x)$ is increasing. (b) In which $f(x)$ is decreasing.

8. A right circular cone is inscribed in a cone. Show that the curved surface area of the cylinder is maximum when the diameter of the cylinder is equal to radius of the base of cone.

9. For the function $f(x) = 2x^3 - 6x^2 - 48x + 17$,find the interval (S):
(a) In which $f(x)$ is increasing. (b) In which $f(x)$ is decreasing.

10. A particle moves along a curve $6y = x^3 + 2$. Find the point on the curve at which y-coordinate is changing 8 times as fast as x- coordinate.

11. A open tank with square base and vertical side to be constructed from a metal sheet so as to hold a given quantity of water. Show that cost of material can be least when the depth of the tank is half of the its with.

12. At what point of ellipse $16x^2 + 9y^2 = 400$,dose the ordinate decreases at the same rate of the abscissa increases.

13. Show that a closed right circular cylinder of given total surface area S amd maximum volume V in such that its is equal to the diameter d of the base.

14. If $y = x^4 - 10$ and if x changes from 2 to 1.97, using differentials, find approximate change in y.

15. A wire of length 25 is to be cut into two pieces. One of two pieces is made into square and other into circle. What should be the length of the pieces so that the combined area of square and circle is minimum?

16. Using differential find the approximate value of $\sqrt{26}$.

17. A window in form of square surmounted by a semicircle opening.if the perimeter of the window is 20m, find the dimension of the window so the maximum possible light is admitted through the whole opening.

18. Find the interval in which the function $f(x)=x^3-6x^2+9x+15$ is increasing or decreasing.
19. Find the largest possible area of right triangle whose hypotenuse is 5 cm long.
20. Find interval in which following function is increasing or decreasing $f(x)=\sin x-\cos x, 0<x<2\pi$.
21. A closed right circular cylinder has a volume of 2156 .cu.cm? what will the radius of its base so that its total surface area is minimum.
22. Find the interval in which function is increasing or decreasing.

    $f(x)=\log(1+x)-\dfrac{x}{1+x}$
23. Find the interval in which the following function is increasing or decreasing.
    $f(x)=(1+x).e^{-x}$.
24. Using differential find the approximate value of $\sqrt{0.37}$.
25. Find the interval in which the function $f(x)=2x^3-9x^2+12x+30$ is (a) increasing (b) decreasing.
26. Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius R is $\dfrac{2R}{\sqrt{3}}$.
27. Using differential find the approximate value of $\sqrt{0.26}$.
28. Using differential find the approximate value of $\sqrt{0.48}$.
29. At what point on the curve $x^2+y^2-2x-4y+1=0$ is a tangent parallel to y-axis?
30. Find the interval in which the function $f(x)=x^3-1`2x^2+36x+17$ is (a) increasing (b) decreasing.
31. An open box with square base is to be made out of given quantity of sheet of area a^2. Show that maximum volume of the box is $\dfrac{a^3}{6\sqrt{3}}$.
32. Using differential find the approximate value of $\sqrt{0.24}$.
33. Find the equation of the tangent to the curve $x^2+3y^2=3$, which is parallel to the line $y-4x+5=0$.
34. Find the interval in which function $f(x)=(x+1)^3(x-3)^3$ is increasing or decreasing?
35. Show that volume of largest cylinder which can be inscribed in a cone of height h and semi-verticle angle 30^0 is $\dfrac{4}{81}\pi h^3$.
36. Show that volume of largest cylinder which can be inscribed in a cone of height h and semi-verticle angle 45^0 is $\dfrac{4}{27}\pi h^3$.

37. Show that the tangent to the curve $y = 2x^3 - 4$ at points $y = 2x^3 - 4x = 2$ *and* $y = -2$ are parallel.
38. Show that volume of largest cylinder which can be inscribed in a cone of height h and semi-verticle angle α is $\frac{4}{27}\pi h^3 \tan\alpha$.
39. Show that the curve $xy = a^2$ *and* $x^2 + y^2 = 2a^2$ touch other.
40. A window in the form of rectangle above which there is a semi-circle. If the perimeter of window is p cm. show that the window will allow the maximum possible light only when radius of semicircle $\frac{p}{\pi + 4}$ cm.
41. If $y = x^4 + 10$ and x changes from 2 to 1.99. find the approximate change in y.
42. Show that the rectangle of maximum area that can be inscribed in a circle of radius r is a square of side $\sqrt{2}r$.
43. Using differential find the approximate value of $\sqrt{0.48}$.
44. Two sides of the triangle have lengths 'a' and 'b' and the angle the between them is θ.what value of θ will maximize the area of triangle? Find the maximum area of triangle also.
45. Show that the function $f(x) = x^3 - 6x^2 + 12x + 8$ is an increasing on R.
46. Find the point on the curve $y^2 = 8x$ for abscissa and ordinate changes at same rate.
47. A balloon which always remains spherical is being inflated by pumping in gas at the rate of $900cm^3/\sec$. Find the rate at which the radius of the balloon is increasing when the radius of the balloon is 15cm.
48. A square piece of side 18 cm.is to be made in to a box without a top by cutting a square piece from each corner and folding up the flaps. What should be the side of square to be cut off , so that volume of box be maximum? Also find the volume of the box.
49. Find the interval on which function $f(x) = \frac{x}{1 + x^2}$ is
    (a) Increasing (b) decreasing
50. Prove that the curve $x = y^2$ and $xy = k$ each other at right angles if $8k^2 = 1$.
51. Prove that volume of the largest cone can be inscribed in a sphere of radius R is $\frac{8}{27}$ of the volume of the sphere.
52. Find the interval in which function $f(x) = x^3 - 6x^2 + 9x + 15$ is
    (a) Increasing (b) decreasing
53. Prove the curve $y = 4x^3 - 2x^5$, find all point at which tangent passes through the origin.

54. A window in form of rectangle surmounted by a semi-circle. If the perimeter of the window is 100m , find the dimension of the window so that maximum light enters through the window.
55. A particle moves along the curve $y=\frac{2}{3}x^3+1$, find the points on the curve at which y- co-ordinate changing twice as fast as x-co-ordinate.
56. Find the equation of the tangent and normal to the curve $x=1-\cos\theta, y=\theta-\sin\theta\ at\ \theta=\frac{\pi}{4}$.
57. Find the interval $f(x)=\frac{4x^2+1}{x}$, x=0 is (a) increasing (b) decreasing .
58. Find equation tangent and normal to the curve $y=x^2+4x+1$at the point whose x-co-ordinate is 3.
59. Find the interval in which the function $f(x)=x^3-6x^2+9x+15$ is (a) increasing (b) decreasing.
60. The volume of spherical balloon is increasing at rate of $25cm^3/\sec$ .find the rate of change of its surface area at the instant when its radius is 5 cm.
61. Find the interval in which the function $f(x)=2x^3-6x^2-48x+17$ is (a) increasing (b) decreasing.
62. Find the equation of the tangent to the curve $y=x^3+2x+6$ which are perpendicular to the line $x+14y+4=0$.
63. Find the interval in which the function $f(x)=x^3-6x^2+9x+15$ is (a) increasing (b) decreasing.
64. Find the interval in which $f(x)=2x^3-9x^2-24x-5$, is (a) increasing (b) decreasing .
65. Using differential find the approximate value of $\sqrt{0.37}$. Correct upto three decimal places.
66. The surface area of a spherical bubble is increasing at the $2cm/\sec^2$. Find the rate at which the volume of the bubble is increasing at the instant of radius is 6 cm.
67. A wire of length 36 cm is cut into two pieces is turned in the form of square and the other into a circle. What should be the lengths of each pieces so that sum of the area of two be minimum.
68. Prove that the line $\frac{x}{a}+\frac{y}{b}=1$is a tangent to the curve $y=be^{-x/a}$,at the point where the curve cuts y-axis.
69. An open box with square base is made to be out of a given iron sheet of area 27 sq.m. Show that the maximum volume that can be inscribed in sphere of radius 12 cm is 16 cm.

70. Find the interval in which the function $f(x)=2x^3-15x^2+36x+1$ is strictly increasing or decreasing. Also find the point on which the tangent are parallel to the x- axis.
71. Show that the height of the cone of maximum volume that can be inscribed in a sphere of radius 12cm. is 16cm.
72. Find the equation of the tangent to the curve $y=\sqrt{3x-2}$ which is parallel to the line $4x-2y+5=0$.
73. Using differential, find the approximate value of $(82)^{1/4}$ upto 3 places of decimal·
74. Find the interval in which the function $f(x)=2x^3-3x^2-36x+7$ is (a) strictly increasing (b) strictly decreasing.
75. Find the equation of tangent and normal to the curve $16x^2+9y^2=144$ at (x_1,y_1) where $x_1=2\, and\, y_1=0$.also find the point of intersection where both tangent and normal cut the X-axis.
76. Find the point on the parabola $f(x)=(x-3)^2$, where the tangent is parallel to the chord joining the point (3,0)and (4,1).
77. Find the equation of line through the point (3,4) which cut from the first quadrant a triangle of minimum area.
78. How that the maximum volume of cylinder which can be inscribed in a sphere of radius $5\sqrt{3}$ is $500\pi cm^3$.
79. A point sources of light along a straight road is a height of 'a' metre. A boy 'b' metre in height is walking on the road. How fast is the shadow increasing if he is walking away from the light at the rate of c metre per minute?
80. The volume of cube is increased at the rate of 7cubic centimeter per second. How fast the surface area of the cube increasing when the length of an edge is 12 centimetre ?
81. Find the equation of the tangent and normal to the curve $y=x^3$ at tha point (1,1).
82. Given the sum of perimeters of a square and a circle prove that the sum of their area least when the side of square is equal to diameter of the circle.
83. Find the equation of tangent to the curve $x=\theta+\sin\theta, y=1+\cos\theta\, at\, \theta=\frac{\pi}{4}$.
84. Prove that the tangent to the curve $y=x^2-5x+6$ at the point P(2,0) and Q(3,0) are at right angle to each other.
85. Find the equation of the line through the point P(3,4) which cuts from first quadrant a triangle of minimum area. Also find the area of triangle.
86. Find the point on the curves $y^2=4x$ which is nearest to the point (2,-8).
87. Prove that the curve $y^2=4ax$ and $xy=c^2$ cut at right angle of $c^4=32a^4$.

88. Show th...

89. Find the points on the curve $y = x^3 - 3x^2 + 2x$ at which tangent to the curve is parallel to line y-2x+3=0.
90. The two equal sides of isosceles triangle with fixed base 6 cm are decreasing at the rate of 3cm/sec. how fast is the area decreasing when the two equal sides are equal to the base.
91. Find the point on the parabola $x^2 = 8y$ which is nearest to the point (2,4) .
92. Prove that $\left(\frac{x}{a}\right)^n + \left(\frac{y}{b}\right)^n = 2$ touches the straight line $\left(\frac{x}{a}\right) + \left(\frac{y}{b}\right) = 2$ for all $n \in N$ at the point (a,b).
93. A wire of length 28m is to be cut into two pieces. One of the two is to be made into a square and other into a circle. What should be the length of two pieces so that the combined area of these two are minimum.
94. Find the point on the curve $y = x^3$ at which slope of the tangent is equal to the Y-coordinate of the point.
95. Find the point on the curve $y = x^3$ where the slope of the tangent is equal to the X -coordinate of the point.
96. A point on the hypotenuse of a triangle is at distances 'a' and 'b' from the sides. Show that the minimum length of hypotenuse is $\left(a^{2/3} + b^{2/3}\right)^{3/2}$.
97. Prove that the volume of the largest cone that can be inscribed in a sphere of radius R is $\frac{8}{27}$ of the volume of sphere.
98. Prove that the curve $x = y^2$ *and* $xy = k$ intersect at right angles if $8k^2 = 1$.
99. Find the maximum area of isosceles triangle inscribed in ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with its vertex at one end of major axis.
100. Show that the semi-vertical angle of the right circular cone of given total surface area and maximum volume is $\sin^{-1}\frac{1}{3}$.
101. Find the equation of tangent to the curve $x = \sin 3t, y = \cos 2t, at\ t = \frac{\pi}{4}$.

Note : if any mistake on this, kindly inform on the mail id : bknal1207@gmail.com

Your Observation! Our Correction !!