MATHS

Assignment 1.0

Differential Equations

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Assignment 1.0

1. Solve the differential equation : $\dfrac{dy}{dx} + \sqrt{\dfrac{1-y^2}{1-x^2}} = 0$

2. A wet porous substance in the open air loses its moisture at a rate proportional to the moisture content. If a sheet hung in the wind loses half its moisture during the first hour, when will it have lost 95% moisture, weather conditions remaining the same.

3. Solve the differential equation : $\dfrac{dy}{dx} + \dfrac{\cos x . \sin y}{\cos y} = 0$

4. A wet porous substance in the open air loses its moisture at a rate proportional to the moisture content. If a sheet hung in the wind loses half its moisture during the first hour, when will it have lost 90% moisture, weather conditions remaining the same ?

5. Form the differential equation of the family of curves represented by the equation : $(x+a)^2 - 2y^2 = a^2$

6. Solve the differential equation : $x\sqrt{1-y^2}\,dx + y\sqrt{1-x^2}\,dy = 0$

7. Solve the following differential equation : $x\dfrac{dy}{dx} + 2y = x\cos x$.

8. Form the differential equation of the family of curves represented by the equation : $(2x+a)^2 + y^2 = a^2$

9. Solve : $(1+x)(1+y^2)dx + (1+y)(1+x^2)dy = 0$

10. The slope of the tangent at any point of a curve is λ times the slope of the straight line joining the point of contact to the origin. Formulate the differential equation representing the problem and hence find the equation of the curve.

11. Solve : $\cos x . \cos y\, dy + \sin x . \sin y\, dx = 0$

12. The normal lines to a given curve at each point (x,y) on the curve pass through the point (2,0). The curve passes through the point (2, 3). Formulate the differential equation representing the problem and hence find the equation to the curve.

13. Solve the differential equation $\dfrac{dy}{dx} = 1 + e^{x-y}$.

14. Solve the differential equation $\dfrac{dy}{dx} = x^2 e^{-3y}$, given that $y = 0$ *for* $x - 0$.

15. Solve the following differential equation : $x^2 dy + y(x+y)dx = 0$.

16. Solve the differential equation : $\dfrac{dy}{dx} = 1 + x + y + xy$.

17. Solve the differential equation $(x + 2y^2)\dfrac{dy}{dx} = y$, given that when x = 2, y = 1.

18. Solve the differential equation $(1+y^2)(1+\log x)dx + xdy = 0$, given that when x =1, y = 1.

19. Show that $y = ae^{2x} + be^{-x}$ is a solution of $\dfrac{d^2y}{dx^2} - \dfrac{dy}{dx} - 2y = 0$.

20. Solve the following differential equation : $x\dfrac{dy}{dx} - y = x + 1$.

21. Solve the following differential equation :

22. Solve : $(x-1)\dfrac{dy}{dx} = 2x^3 y$

23. Solve : $x\dfrac{dy}{dx} = y(\log y - \log x - 1)$

24. Solve : $(x+2)\dfrac{dy}{dx} = 4x^2 y$

25. Solve : $x.\dfrac{dy}{dx} = y - x.\tan\left(\dfrac{y}{x}\right)$.

26. Solve the differential equation : $\cos x(1 + \cos y)dx - \sin y(1 + \sin x)dy = 0$

27. Solve $\dfrac{dy}{dx} - 3y\cot x = \sin 2x$, given that y = 2 when $x = \dfrac{\pi}{2}$.

28. Solve the differential equation : $(1 + y^2)dx = (\tan^{-1} y - x)dy$

29. Solve the differential equation : $ydy - (1 - x^2 - y^2)xdx = 0$.

30. Solve the following differential equation : $\left(y - x\dfrac{dy}{dx}\right) = a\left(y^2 + \dfrac{dy}{dx}\right)$

31. Solve the following differential equation : $(1 + x^2)\dfrac{dy}{dx} + y = \tan^{-1} x$

32. Solve the following differential equation : $(y + xy)dx + (x - xy^2)dy = 0$

33. Solve the following differential equation : $\dfrac{dy}{dx} - y = x.e^x$

34. Solve the following differential equation : $(x^3 + y^3)dy - x^2 ydx = 0$.

35. Solve the following differential equation : $x\dfrac{dy}{dx} - y = x^2$.

36. Solve : $\dfrac{dy}{dx} + 2y = xe^{4x}$.

37. Solve : $\dfrac{dy}{dx} = \sqrt{4 - y^2}$.

38. Solve : $\dfrac{dy}{dx} = 1 - x + y - xy$.

39. Form the differential equation of the following family of curves : $xy = Ae^x + Be^{-x} + x^2$

40. Solve the following differential equation : $x^2\dfrac{dy}{dx} = 2xy + y^2$.

41. Solve the following differential equation : $\cos^2 x\dfrac{dy}{dx} + y = \tan x$.

42. Solve the following differential equation : $\frac{dy}{dx} + \frac{1}{x}y = y^3$.

43. Solve the following differential equation : $\frac{dy}{dx} + \frac{1}{x} = \frac{e^y}{x}$.

44. Solve the following differential equation : $(1+x)\frac{dy}{dx} + 1 = e^{x-y}, x \neq -1$.

45. Find the differential equation of the family of curves given by $x^2 + y^2 = 2ax$.

46. Solve the differential equation, given that y = 1 when x = 2 : $x\frac{dy}{dx} + y = x^3$.

47. Solve the following differential equation : $(1+y^2)(1+\log x)dx + xdy = 0$.

48. Solve the following differential equation : $\frac{dy}{dx} - \frac{1}{x}.y = 2x^2$.

49. Show that the differential equation of all parabolas $y^2 = 4a(x-b)$ is given by $y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 0$.

50. Solve the following differential equation : $(1+e^{2x})dy + (1+y^2)e^x dx = 0$.

51. Solve the following differential equation : $2x\frac{dy}{dx} + y = 6x^3$.

52. Form the differential equation corresponding to $y^2 = a(b-x^2)$, where a and b are arbitrary constants.

53. Solve the following differential equation : $x\frac{dy}{dx} - y - 2x^3 = 0$.

54. Solve the following differential equation : $(y+x)\frac{dy}{dx} = y - x$.

55. Solve the following differential equation : $\frac{dy}{dx} = y\sin 2x$, given that y(0) = 1.

56. Solve the following differential equation : $x\frac{dy}{dx} - ay = x+1$.

57. Solve the following differential equation : $(x^2 - yx^2)dy + (y^2 + x^2y^2)dx = 0$.

58. Solve the following differential equation : $\frac{dy}{dx} + 2y\tan x = \sin x$.

59. Solve the following differential equation : $\frac{dy}{dx} + y\cot x = 2\cos x$.

60. Solve the following differential equation : $(3xy + y^2)dx + (x^2 + xy)dy = 0$.

61. Solve the following differential equation : $\frac{dy}{dx} + y\cot x = x^2\cot x + 2x$.

62. Solve the following differential equation : $(1+x^2)\frac{dy}{dx} - 2xy = (x^2+2)(x^2+1)$.

63. Form the differential equation representing the family of curves $y^2 - 2ay + x^2 = a^2$ where a is an arbitrary constant.

64. Solve the following differential equation : $\dfrac{dy}{dx} + y \cot x = x^2 \cot x + 2x$.

65. Solve the following differential equation : $(1 + x^2)\dfrac{dy}{dx} - 2xy = (x^2 + 2)(x^2 + 1)$.

66. Solve the following differential equation : $x\dfrac{dy}{dx} - y = \sqrt{x^2 + y^2}$

67. Solve the following differential equation : $\left(x^2 + xy\right)dy = (x^2 + y^2)dx$

68. Solve the following differential equation : $\dfrac{dy}{dx} + \dfrac{y}{x} = e^x : x > 0$

69. Solve the following initial value problem : $2x^2\dfrac{dy}{dx} - 2xy + y^2 = 0; y(e) = e$.

70. Solve the following differential equation : $2xy + y^2 - 2x^2\dfrac{dy}{dx} = 0; y(1) = 2$.

71. Solve the following differential equation : $x(1 + y^2)dx - y(1 + x^2)dy = 0$, given that $y = 0$ when x = 1.

72. Solve the following differential equation :

73. Form the differential equation of the family of curves $y = a\sin(x + b)$, where a and b are arbitrary constants.

74. Solve the following differential equation : $2xy\,dx + (x^2 + 2y^2)dy = 0$.

75. Solve the following differential equation : $\dfrac{dy}{dx} + \sec x . y = \tan x$.

76. Solve the following differential equation : $\sin x\dfrac{dy}{dx} + y\cos x = \cos x \sin^2 x$.

77. Solve the following differential equation : $(y^2 - x^2)dy = 3xy\,dx$.

78. Verify that $y = A\cos x - B\sin x$ is a solution of the differential equation $\dfrac{d^2y}{dx^2} + y = 0$.

79. Solve the following differential equation : $\dfrac{dy}{dx} = \log(x + 1)$.

80. Solve the following differential equation : $\dfrac{dy}{dx} - \dfrac{y}{x} = \left(\dfrac{x-1}{x}\right)e^x$.

81. Verify that $y = e^x \cos x$ is a solution of the differential equation $\dfrac{d^2y}{dx^2} - 2\dfrac{dy}{dx} + 2y = 0$.

82. Solve the following differential equation :

83. Form the differential equation of the family of curves $y = a\cos(x + b)$, where a and b are arbitrary constants.

84. Solve the following differential equation : $4\dfrac{dy}{dx} + 8y = 5e^{-3x}$

85. Solve the differential equation : $\sec^2 x \tan y\, dx + \sec^2 y \tan x\, dy = 0.$

86. Solve the following differential equation : $x\cos y\, dy = (xe^x \log + e^x)dx.$

87. Solve the following differential equation : $y(1 - x^2)\dfrac{dy}{dx} = x(1 + y^2).$

88. Form the differential equation of the family of curves $y = A\cos 2x + B\sin 2x$, where A and B are constants.

89. Solve the following differential equation : $\dfrac{dy}{dx} + 2y = 6e^x.$

90. If $y = Ae^{mx} + Be^{nx}$, prove that $\dfrac{d^2 y}{dx^2} - (m+n)\dfrac{dy}{dx} + mny = 0.$

91. Form the differential equation for all those ellipses having foci on Y-axis whose centres lie at the origin.

92. Solve the following differential equation : $x^2 y\, dx - (x^3 + y^3)dy = 0.$

93. Solve the following differential equation : $\cos x\dfrac{dy}{dx} + y = \sin x.$

94. Solve the following differential equation : $(x^2 + xy)dy + (3xy + y^2)dx = 0.$

95. Solve the following differential equation : $(y^2 - x^2)dy - 3xy\, dx = 0.$

96. Find the differential equation of the family of circles passing through the origin with centres on y-axis.

97. Solve the following differential equation :

98. Form the differential equation representing the parabolas having vertex at the origin and axis along positive direction of x-axis.

99. Solve the following differential equation : $(3xy + y^2)dx + (x^2 + xy)dy = 0.$

100. Solve the following differential equation : $(1 + x^2)\dfrac{dy}{dx} + y = \tan^{-1} x.$

101. Solve the following differential equation : $x\log x\dfrac{dy}{dx} + y = 2\log x$

102. Solve the following differential equation : $(x^2 - y^2)dx + 2xy\, dy = 0$ given that y = 1 when x = 1.

103. Solve the following differential equation : $\dfrac{dy}{dx} = \dfrac{x(2y - x)}{x(2y + x)}$, if y = 1 when x = 1.

104. Solve the following differential equation : $\cos^2 x\dfrac{dy}{dx} + y = \tan x.$