IIT-JEE / ENGINEERING

MATHS

FORMULA

VECTOR

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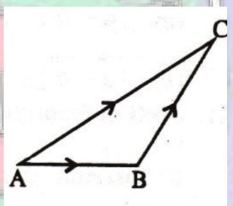
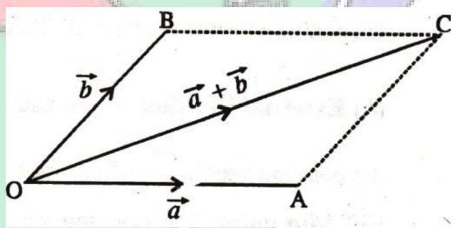
IMPORTANT DEFINATIONS, FORMULAE AND METHODS

1. **Scalars:** These are the physical quantities which have only magnitude, but no direction.
2. **Vectors:** These are the physical quantities which have both magnitude and direction.
3. **Directed Line Segment:** Any given portion of a straight line, for which the two end points are distinguished as initial and terminal points is called a **directed line segment.**
   Note: A directed line segment has magnitude as well as direction.
4. **Position Vector of a point :** The position vector of a point P with respect to a fixed point 'O' is the vector $\overrightarrow{OP}$. The fixed point 'O' is called the origin.
   Note : (a) A given vector can be expressed as position vector of the terminal point minus position vector of the initial point.
   (b)Position vector of point $P(x, y, z)$ is $x\hat{i}+y\hat{j}+z\hat{k}$.
   (c) Magnitude of position vector of point $P(x, y, z)$ is , $|\overrightarrow{OP}| = \sqrt{x^2+y^2+z^2}$
   (d) Component vector of $\overrightarrow{OP}$ along X-axis is the vector $x\hat{i}$.
   (e) Component vector of $\overrightarrow{OP}$ along Y-axis is the vector $y\hat{j}$.
   (f) Component vector of $\overrightarrow{OP}$ along Z-axis is the vector $z\hat{k}$.
   (g) x, y, and z are called scalar composition or rectangular components.
   (h) $x\hat{i}$, $y\hat{j}$ and $z\hat{k}$ are called vector components.
5. **Direction cosines.** (a) The angles made by positive vector $\overrightarrow{OP}$ $(\vec{r})$ with the positive directions of x, y and z-axes respectively are called direction angles.
   (b)The cosine values of above angle are called direction cosines of vector $\vec{r}$.
   (c) Direction cosines are denoted by l, m and n.
   (d) The coordinated of the point P can be expressed as (lr, mr, nr).
   (e) The numbers proportional to direction cosines are called direction ratios.
   (f) Direction ratios are denoted by a, b, c.
   (g) $a = lr$, b = mr, c = nr
   (h) $l^2+m^2+n^2=1$ but $a^2+b^2+c^2 \neq 1$
6. **Zero Vector OR Null Vector :** A vector whose initial and terminal points are coincident is called the zero vector. It is denoted as $\vec{O}$. It has zero magnitude but and has arbitrary direction.
7. **Unit Vector :** A unit vector of a given vector is a vector of magnitude one unit and direction that of a given vector. It is denoted as $\hat{a}$ for a given vector $\vec{a}$.
   Note : Unit vector, $\hat{a} = \dfrac{\vec{a}}{|\vec{a}|}$
8. **Coinitial Vectors :** Two or more vectors having same initial point are called coinitial vectors.

9. **Parallel Vectors :** Two or more vectors are said to be Parallel vectors if they have the same or parallel supports.
   Note : Parallel vectors may have unequal magnitudes and opposite senses.
10. **Collinear Vectors :** Two or more vectors are said to be collinear if they have same or parallel support.
11. **Like Vectors :** Two parallel vectors having the same direction are called like vectors.
12. **Unlike Vectors :** Two parallel vectors having the opposite direction are called like vectors.
13. **Free Vector :** A vector whose initial point is not specified is called a free vector.
14. **Localized Vector:** A vector drawn parallel to a given vector through a specified point as the initial point is called a localized vector.
15. **Negative of a Vector :** A vector whose magnitude is the same as that of given vector, but direction is opposite to that of it, is called negative of the given vector.
16. **Equal Vectors :** Two vector $\vec{a}$ and $\vec{b}$ are said to be equal if they have the
   (a) Same magnitude
   (b) Same direction (i.e., sense)
   (c) Same support (or parallel support).
17. **Triangle law of vector addition :** If two vectors are represented in magnitude and direction by the sides of a triangles taken in order then their sum is represented in magnitude and direction by the third side of the triangle taken in opposite order.



   $\overrightarrow{AB}+\overrightarrow{BC}=\overrightarrow{AC}$
   Note : In ΔABC,
   $\overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CA}=\vec{0}$
18. **Parallelogram law of vector addition :** If two vectors $\vec{a}$ and $\vec{b}$ represented by the two adjacent sides of a parallelogram in magnitude and direction then their sum $\vec{a}+\vec{b}$ is represented in magnitude and direction by the parallelogram through their common point. This is known as parallelogram law of vector addition.



19. **Multiplication of a vector by a scalar :** Let $\vec{a}$ be any given vector and m given scalar. Then the product of the $\vec{a}$ by the scalar m is called scalar multiplication.
20. **Properties of vector addition and scalar multiplication :**
   (a) $\vec{a}+\vec{b}=\vec{b}+\vec{a}$ (Commutativity)
   (b) $\vec{a}+(\vec{b}+\vec{c})=(\vec{a}+\vec{b})+\vec{c}$ (Associativity)
   (c) $\vec{a}+\vec{0}=\vec{0}+\vec{a}=\vec{a}$ ($\vec{0}$ is called additive identity)
   (d) $\vec{a}+(-\vec{a})=(-\vec{a})+\vec{a}=\vec{0}$ ($-\vec{a}$ is called additive inverse of $\vec{a}$)

(e) $\vec{a}-\vec{b}=\vec{a}+(-\vec{b})$

(f) $m(\vec{a}+\vec{b})=m\vec{a}+m\vec{b}$

(g) $(m+n)\vec{a}=m\vec{a}+n\vec{a}$

(h) $0\vec{a}=\vec{0}$

(i) $(-1)\vec{a}=-\vec{a}$

(j) $m(-\vec{a})=(-m)\vec{a}=-(m\vec{a})$

(k) $(-m)(-\vec{a})=m\vec{a}$

(l) $m(n\vec{a})=(mn)\vec{a}$

(m) $|\vec{a}+\vec{b}|\le|\vec{a}|+|\vec{b}|$ (triangle inequality)

(n) $|\vec{a}-\vec{b}|\le|\vec{a}|+|\vec{b}|$

(o) $|\vec{a}-\vec{b}|\ge|\vec{a}|-|\vec{b}|$

21. Section formula :

(i)Internal Division : Let A and B be two points with position vectors $\vec{a}$ and $\vec{b}$ respectively. Then the positive vector $\vec{r}$ of a point P dividing AB internally in the ratio m:n is given by $\vec{r}=\dfrac{m\vec{b}+n\vec{a}}{m+n}$

(ii) External Division : Let A and B be two points with position vectors $\vec{a}$ and $\vec{b}$ respectively. Then the positive vector $\vec{r}$ of a point P dividing AB externally in the ratio m:n is given by $\vec{r}=\dfrac{m\vec{b}-n\vec{a}}{m-n}$.

(iii)Mid point : The position vector of the midpoint of the join of two points with positive vectors $\vec{a}$ and $\vec{b}$ is $\dfrac{1}{2}(\vec{a}+\vec{b})$.

22. Condition for Collinear Vectors : Two vectors $\vec{a}$ and $\vec{b}$ are collinear if and only if there exists a non zero scalar such that $\vec{a}=\lambda\vec{b}$.

In terms of components, two vectors $\vec{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$ and $\vec{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$ are collinear if and only if $\dfrac{a_1}{b_1}=\dfrac{a_2}{b_2}=\dfrac{a_3}{b_3}=\lambda$

23. Scalar or (dot) product of two vectors : Let $\vec{a}$ and $\vec{b}$ be any two vectors and θ be any angle between them. Then the scalar product of vector $\vec{a}$ and $\vec{b}$ is denoted by $\vec{a}\,.\,\vec{b}$ and is defined as $\vec{a}\,.\,\vec{b}=|\vec{a}||\vec{b}|\cos\theta$

Note : (a) $\vec{a}\,.\,\vec{b}$ being a real number is scalar.

(b) $\cos\theta=\dfrac{\vec{a}\,.\,\vec{b}}{|\vec{a}||\vec{b}|}$

(c) If $\vec{a}=\vec{0}$ or $\vec{b}=\vec{0}$ then $\vec{a}\,.\,\vec{b}=0$

(d) If $\vec{a}$ and $\vec{b}$ are like vector then $\vec{a}\,.\,\vec{b} = |\vec{a}||\vec{b}|$ $[\because \theta = 0]$

(e) If $\vec{a}$ and $\vec{b}$ are unlike vector then $\vec{a}\,.\,\vec{b} = -|\vec{a}||\vec{b}|$ $[\because \theta = \pi]$

(f) If $\vec{a}$ and $\vec{b}$ are perpendicular to each other then $\vec{a}\,.\,\vec{b}=0$ $\left[\because \theta = \frac{\pi}{2}\right]$

(g) For orthonormal vector triad $\hat{i}, \hat{j}, \hat{k}$

$\hat{i}.\hat{i} = \hat{j}.\hat{j} = \hat{k}.\hat{k} = 1$

$\hat{i}.\hat{j} = \hat{j}.\hat{k} = \hat{k}.\hat{i} = 0$

(h) In terms of components, for vectors $\vec{a} = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$ and $\vec{b} = a_2\hat{i} + b_2\hat{j} + c_2\hat{k}$

$$\vec{a}\,.\,\vec{b} = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + a_2^2 + a_3^2}\sqrt{b_1^2 + b_2^2 + b_3^2}}$$

(i)For perpendicular vector $\vec{a}$ and $\vec{b}$

$a_1a_2 + b_1b_2 + c_1c_2 = 0$

(j) For parallel vectors $\vec{a}$ and $\vec{b}$

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

24. Properties of scalar product :

(a) $\vec{a}\,.\,\vec{b} = \vec{b}\,.\,\vec{a}$ (commutative)

(b) $\vec{a}\,.\,(\vec{b} + \vec{c}) = \vec{a}\,.\,\vec{b} + \vec{a}\,.\,\vec{c}$

(c) $(k\,\vec{a}).\,\vec{b} = k(\vec{a}\,.\,\vec{b}) = \vec{a}.(k\,\vec{b})$

25. Projection of a vector along a directed line : Projection of vector $\vec{a}$ on $\vec{b} = \dfrac{\vec{a}\,.\,\vec{b}}{|\vec{b}|}$

26. Vector (or cross) product of two vector : Let $\vec{a}$ and $\vec{b}$ are any two vectors and θ be any angle between them. Then the vector product of vectors $\vec{a}$ and $\vec{b}$ is denoted by $\vec{a} \text{ X } \vec{b}$ and is defined as $\vec{a} \text{ X } \vec{b} = |\vec{a}||\vec{b}|\sin\theta\,\hat{n}$ here $\hat{n}$ is a unit vector perpendicular to both vectors $\vec{a}$ and $\vec{b}$.

Note :

(a) $\vec{a} \text{ X } \vec{b}$ is a vector

(b) $\sin\theta = \dfrac{|\vec{a} \text{ X } \vec{b}|}{|\vec{a}|\ |\vec{b}|}$

(c) If $\vec{a}=\vec{0}$ or $\vec{b}=\vec{0}$ then $\vec{a} \text{ X } \vec{b} = \vec{0}$

(d) For parallel (or Collinear) vectors $\vec{a}$ and $\vec{b}$

$\vec{a}\ X\ \vec{b} = \vec{0}$

(e) $\vec{a}\ X\ \vec{a} = \vec{0}$ and $\vec{a}\ X\ (-\vec{a}) = \vec{0}$

(f) For $\theta = \frac{\pi}{2}$, $\vec{a}\ X\ \vec{b} = |\vec{a}||\vec{b}|\hat{n}$ and for $\theta = \frac{3\pi}{2}$, $\vec{a}\ X\ \vec{b} = |\vec{a}||\vec{b}|(-\hat{n})$

(g) For othonormal vector triad $\hat{i}, \hat{j}, \hat{k}$

$\hat{i}\ X\ \hat{i} = \hat{j}\ X \hat{j} = \hat{k}\ X\ \hat{k} = 0$

$\hat{i}\ X\ \hat{j} = \hat{k}$, $\hat{j}\ X \hat{k} = \hat{i}$, $\hat{k}\ X\ \hat{i} = \hat{j}$

(h) In terms of components for vectors

$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

$$\vec{a}\ X\ \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

(i) A unit vector perpendicular to each of given vectors $\vec{a}$ and $\vec{b}$ is $\pm\frac{(\vec{a}\ X\ \vec{b})}{|\vec{a}\ X\ \vec{b}|}$

27. Properties of cross product

(a) $\vec{a}\ X\ \vec{b} = -\vec{b}\ X\ \vec{a}$

(b) $(m\vec{a})\ X\ \vec{b} = m(\vec{a}\ X\ \vec{b}) = \vec{a}\ X\ (m\ \vec{b})$

(c) $\vec{a}\ X\ (\vec{b} + \vec{c}) = \vec{a}\ X\ \vec{b} + \vec{a}\ X\ \vec{c}$

(d) $\vec{a}\ X\ (\vec{b} - \vec{c}) = \vec{a}\ X\ \vec{b} - \vec{a}\ X\ \vec{c}$

28. Area of Triangle : If $\vec{a}$ and $\vec{b}$ are the adjacent sides of a triangle then area of triangle is given by $\frac{1}{2}|\vec{a}\ X\ \vec{b}|$.

29. Area of a Parallelogram : If $\vec{a}$ and $\vec{b}$ are the adjacent sides of a parallelogram then area of parallelogram is given by $|\vec{a}\ X\ \vec{b}|$.

30. Langranges Indentity : For any vectors $\vec{a}$ and $\vec{b}$, $|\vec{a}\ X \vec{b}|^2 = |\vec{a}|^2\ |\vec{b}|^2 - (\vec{a}\ .\ \vec{b})^2$

Note : if any mistake on this, kindly inform on the mail id :

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