IIT-JEE / ENGINEERING

MATHS

FORMULA

Linear Programming

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IMPORTANT DEFINATIONS, FORMULAE AND METHODS

1. **Open half plane :** The linear inequality ax + by + c > 0 or < 0 is said to be an open half plane when the line ax + by + c = 0 divides the plane into two parts and exactly one of these parts (excluding the line) will be the graph of the given inequality.
2. **Closed half plane :** The linear inequality ax + by + c > 0 or < 0 is said to be a closed half plane when the line ax + by + c = 0 divides the plane into two parts and exactly one of these parts (including the line) will be the graph of the given inequality.
3. **Objective function :** The linear function z = ax + by, where a and b are constants, which is to be optimized (maximized or minimized) is called the objective function.
4. **Constraints :** The linear inequalities on the variables x and y are known as constraints.
5. **Linear programming Problem :** The problem which deals with the optimization (maximization or minimization) of a linear function (called objectives function) of a number of variables x,y subject to a number of condition on these variables (called constraints) is called linear programming problem.
6. **Feasible Region :** The common region determined by all the constraints and non negative constraints $x \geq 0$, $y \geq 0$ of a linear programming problem is called the feasible region.
7. **Infeasible Region :** The region other than the feasible region is known as infeasible region.
8. **Feasible Solutions :** Points inside the feasible region or on the boundary of the feasible region are known as feasible solutions of the constraints.
9. **Infeasible Solutions :** Points outside the feasible region are known as infeasible solutions.
10. **Optimal Solutions :** Any feasible solution which maximizes or minimizes the objectives function is called an optimal solution.

Fundamental Theorem 1. Let R be the feasible region (convex polygon) for a linear programming problem and let Z = ax + by be the objectives functions. When Z has an optimal value (maximum or minimum) where the variables x and y are subject to constraints described by linear inequalities, this optimal value must occur at a corner point (vertex) of the feasible region.

Fundamental Theorem 2. Let R be the feasible region for a linear programming problem, and let Z = ax + by be the objectives function. If R is **bounded** then the objective function Z has both a **maximum** and **minimum** value on R and each of these occurs at corner point (vertex) of R.

If the feasible region is unbounded, then a maximum or minimum may not exist. However, if it exists it must occur at a corner points of R.

Corner or Extreme Point Method :

Step 1. Formulate the linear programming problem in x and y with given conditions.

Step 2. Drawing of the graph : Convert the inequality constraints into equality constraints and then plot each equation as a straight line on the graph paper.

Step 3. Identification of the feasible solution : Find the feasible region by taking a test point and determine its corner points either by inspection or by solving the two equations of the lines intersecting at that point.

Step 4. To get Optimum solution : Evaluate the value of objective function 'Z' at each corner point.

Let M and m be the greatest and smallest values of the objective function Z.

(i) **When the feasible region is bounded :** M and m respectively are the maximum and minimum values of objective function Z.

(ii) **When the feasible region is unbounded :** (i) M is the maximum value of the objectives functions 'Z' if the open half plane determined by ax + by > M has no point in common with the feasible region.

Otherwise the objective function has no maximum value.

(iii) m is the minimum value of the objective function 'Z' if the open half plane determined by ax + by < m has no point in common with the feasible region. Otherwise **the objective** function has no minimum value.

Note : If two corner points of the feasible region are both optimal solutions of the same type i.e. both produce the same maximum or minimum then any point on the line segment joining these two points is also an optical solution of the same type.

Note : if any mistake on this, kindly inform on the mail id :
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