IIT-JEE / ENGINEERING

MATHS

FORMULA

Probability

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IMPORTANT DEFINATIONS, FORMULAE AND METHODS

1. **Introduction :** In earlier classes, we have studied that if A is any event related to an experiment, then

$$P(A) = \frac{n(A)}{n(S)}$$, where

n(A) = number of cases that favour the event A.

n(S) = number of total outcomes of the experiment.

P(A) = Probability of the event A, where $0 \le P(A) \le 1$.

We have also studied the addition theorem for two or more events which is as follow:

If A and B are two events then $P\ (A\ or\ B) = P(A) + P(B) - P(A \cap B)$ and if A and B are mutually exclusive events, then $P\ (A\ or\ B) = P(A) + P(B)\ \{\because A \cap B = \phi\}$

In this chapter, we shall discuss the concept of conditional probability of an event, Bayes' theorem, multiplication rule of probability and independence of events. We shall also learn an important concept of random variable and its probability distribution, mean and variance of probability distribution and binomial distribution.

2. **Conditional Probability :** If E and F are two events associated with the same sample space of a random experiment, then the conditional probability of the events E given that F has occurred, is given by

$$P(E/F) = \frac{P(E \cap F)}{P(F)}\ provided\ P(F) \neq 0$$

$$= \frac{n(E \cap F)}{n(F)} = \frac{\text{Number of elementary events favourable to event } E \cap F}{\text{Number of elementary events favaurable to event F}}$$

3. **Properties of Conditional probability :**

(i) $P(S/F) = P(F/F) = 1$

(ii) If A and B are any two events of sample space S and F is an event of S such that $P(F) \neq 0$, then

$$P[(A \cup B)/F] = P(A/F) + P(B/F) - P((A \cap B)/F)$$

(iii) In particular, if A and B are disjoint events then $P[(A \cup B)/F] = P(A/F) + P(B/F)$

(iv) $P(\bar{E}/F) = 1 - P(E/F)$

4. Multiplication Theorem on Probability :

From the concept of conditional probability, we have

$$P(E/F) = \frac{P(E \cap F)}{P(F)}, P(F) \neq 0$$

$\Rightarrow$ $P(E \cap F) = P(F) . P(E/F)$ ......(1)

Also $P(F/E) = \frac{P(E \cap F)}{P(E)}, P(E) \neq 0$

$\Rightarrow$ $P(E \cap F) = P(E) P(F/E)$ ....(2)

(1) and(2) $\Rightarrow P(E \cap F) = P(E) P(F/E)=P(F) P(E/F)$, provided P(E)and P(F) $\neq 0$

This result is known as the multiplication rule of probability. If we have more than two events E,F and G, then by multiplication rule of probability

$P(E \cap F \cap G) = P(E)P(F/E)P(G/EF)$ where $EF = E \cap F$

Note : When things are drawn without replacement, then we use the above rule to find the required probability.

5. Independent Events : Two events are independence events if the probability of occurrence of one of them is not affected by occurrence of the other.

If E and F are two independent events, then

$P(E/F) = P(E), P(F) \neq 0$

.......(1)

and $P(F/E) = P(F), P(E) \neq 0$

.......(2)

Now by multiplication rule of probability

$$P(E \cap F) = P(E)P(F/E)$$
$$= P(F)P(E/F)$$

.......(3)

Using (1) and (2) in (3), we have

$$P(E \cap F) = P(EF) = P(E).P(F)$$

Hence events E and F are independent event if

$$P(EF) = P(E).P(F)$$

For two independent events E and F, the addition theorem becomes

$$P(E \text{ or } F) = P(E) + P(F) - P(E)P(F)$$

or $$P(E \text{ or } F) = 1 - P(\bar{E}\bar{F}) = 1 - P(\bar{E})\,P(\bar{F})$$

here $\bar{E}$ and $\bar{F}$ are also independent event. Similarly for three events A,B and C.

$$P(A \text{ or } B \text{ or } C) = 1 - P(\bar{A}\,\bar{B}\,\bar{C}) = 1 - P(\bar{A})\,P(\bar{B})\,P(\bar{C})$$

Two events E and F dependents if $P(EF) \neq P(E)\,P(F)$

6. **Partition of sample Space :** The events $E_1, E_2, \ldots\ldots\ldots E_n$ represent a partition of the sample space S if they are pairwise disjoint, exhaustive and have non-zero probabilities.

   In other words, a set of events $E_1, E_2, \ldots\ldots\ldots E_n$ is said to represent a partition of the sample space S if the following hold :

   (a) $E_i \cap E_j = \phi,\ i \neq j$, i, j = 1, 2, 3.........n

   (b) $E_1 \cup E_2 \cup \ldots\ldots \cup E_n = S$ and

   (c) $P(E_i) > 0$ for all i = 1, 2, ...n

7. **Theorem of Total Probability :** Let $\{E_1, E_2, \ldots\ldots\ldots E_n\}$ be a partition of the sample space S and suppose that each of the events $E_1, E_2, \ldots\ldots\ldots E_n$ have non-zero probability of occurrence. Let A be any event associated with S, then

$$P(A) = P(E_1)\,P(A/E_1) + P(E_2)\,P(A/E_2) + \ldots\ldots + P(E_n)\,P(A/E_n)$$

$$= \sum_{j=1}^{n} P(E_j)P(A/E_j)$$

8. **Bayes' Theorem :** If $E_1, E_2, \ldots\ldots\ldots E_n$ are n non empty events which constitute a partition of sample space S, then

$$P(E_i/A) = \frac{P(E_i)P(A/E_i)}{\sum_{j=1}^{n} P(E_j)P(A/E_j)} \text{ for any } i = 1, 2, 3 \ldots\ldots\ldots n.$$

9. **Random Variables and its Probability Distribution :** A random variable is a real valued function whose domain is the sample space of a random experiment.

For example, let us consider the experiment of drawing two cards from 52 cards at random. Let x denotes the number of aces obtained, then x is the random variable and for each outcomes, its value is as given below :

x = 0 , if both the cards drawn are non ace cards

x = 1, if one of the cards drawn is an ace and the other is a non ace card

x = 2, if both the cards drawn are ace cards.

Hence X takes the values 0, 1, 2 here.

The probability distribution of a random variable x is the system of numbers.

x	x_1	x_2	....	x_n
$P(x)$	P_1	P_2	....	P_n

where $P_i > 0, \sum_{i=1}^{n} P_i = 1$, i = 1, 2 ......, n

The real numbers $x_1, x_2, ... x_n$ are the possible values of the random variable x and P_i(i = 1, 2, ..., n) is the probability of the random variable x taking the value x_i i.e.

$$P(x = x_i) = P_i$$

10. Mean of a Random Variable : The mean of a random variable x is also called expectation of x , denoted by E(x) is given by

$$E(x) = \mu = \sum_{i=1}^{n} x_i P_i = x_1 p_1 + x_2 p_2 + ... + x_n p_n$$

where $x_1, x_2, ... x_n$ are the possible values of the random variable x with probabilities $p_1, p_2, ... p_n$ respectively.

11. Formula to find the Variance of a Random Variable : Let x be a random variable with values x = $x_1, x_2, ... x_n$ and their respective probabilities are $p_1, p_2, ... p_n$, then $Var(x) = E(x^2) - (E(x))^2$

where $E(x^2) = \sum_{i=1}^{n} x_i^2 p(x_i)$

and E (x) = μ = mean of x.

The standard deviation of x i.e. S.D. (x) = $\sigma_x = \sqrt{\text{var}(x)}$

12. Bernoulli Trials : Trials of a random experiment are called Bernoulli trials, if they satisfy the following conditions :

(i) There should be a finite number of trials.

(ii)The trials should be independent.

(iii) Each trail has exactly two outcomes : success or failure.

(iv) The probability of success remains the same in each trial.

13. Binomial Distribution : The probability distribution of number of success in an experiment consisting of n Bernoulli trials may be obtained by the binomial expansion of $(q + p)^n$, where p = probability of success and

q = probability of failure and p + q = 1

The binomial distribution of number of success x is given in the following table:

x	0	1	2	....	n
P(x)	${}^nC_0q^n$	${}^nC_1q^{n-1}p^1$	${}^nC_5q^{n-2}p^2$		${}^nC_np^n$

Here n and p are the parameters of binomial distribution.

The probability of x success P(X = r) = ${}^nC_rq^{n-r}p^r$,

r = 0, 1,........ n and p + q = 1

For a binomial distribution mean of x = np and var (x) = npq

Note : if any mistake on this, kindly inform on the mail id :

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