IIT-JEE / ENGINEERING

MATHS

FORMULA

Relations and Functions

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IMPORTANT DEFINATIONS, FORMULAE AND METHODS

1. **Ordered pair** : Let A and B are two non-empty sets, then (a, b) is called an ordered pair if and only if $a \in A$ and $b \in B$.
2. **Cartesian product of two sets :** Let A and B are two non-empty sets. Then the cartesian product of two sets A and B is the set of all ordered pairs (a, b) $\forall\ a \in A$ and $b \in B$. It is written as A X B
   $\therefore A\ X\ B = \{(a,b) :\ a \in A, b \in B\}$
3. **Relation :** Let A and B are two non-empty sets, then any subset of A X B is called a relation. It is denoted by symbol R.
   Note : (*a*) (a, b) $\in$ R can be read as *a* is related to b and is denoted as *a* R b.
   (b) It is also called binary operation.
   (c) The number of relations on set $A = 2^{n(AXA)} = 2^{n^2}$
4. **Empty Relation :** A relation R on set *a* set A is called an empty relation, if no element of A is related to any element of A.
   Therefore $\phi \subset A\ X\ A$ is called a void or an empty relation on A.
5. **Universal Relation :** A relation R on a set A is called universal relation, if each element of A is related to every element of A.
   Therefore, $A\ X\ A \subset A\ X\ A$ is called a universal relation on A.
6. **Identity Relation :** Let A be a non – empty set. Then a relation I_A on A is called an identity relation if and only if the image of $a \in A$ on I_A is $a\ \forall\ a \in A$.
   $i.e.\ \mathrm{I_A} = \{(a,a) : a \in A\}$
7. **Inverse Relation :** Let $R = \{(a,b) : a,b \in A\}$ be a relation on A. Then a relation $\{(b,a) : a,b \in A\}$ is called an inverse of relation R and is denoted by R^{-1}.
   $\therefore R^{-1} = \{(b,a) : b \in B,\ a \in A,\ (a,b) \in R\}$
8. **Reflexive Relation :** A relation R on a set A is said to be reflexive, if every element of A is related to itself.
   Note : (1) R is reflexive $\Leftrightarrow (a,a) \in R\ \forall\ a \in A$
   (2) R is not reflexive, if there exists an element $a \in A$ such that $(a,a) \notin R$
9. **Symmetric Relation :** A relation R on set A is said to be symmetric, if $(a,b) \in R \Rightarrow (b,a) \in R\ \forall\ a,b \in A$.
10. **Transitive Relation :** A relation R on set A is said to be transitive, if $(a,b) \in R$ and $(b,c) \in R \Rightarrow (a,c) \in R\ \forall\ a,b,c \in A$.
11. **Equivalence Relation :** A relation R on a set A is said to be an equivalence relation if and only if R is reflexive, symmetric and transitive.

12. **Domain, Co-domain and Range of a Relation :** Let A and B are two non-empty sets and R be a relation from A to B. Then

(i) Domain of relation R is the set of all first elements of ordered pairs $(a,b) \in R$

$\therefore D_R = \{a : a \in A, (a,b) \in R\}$

(ii) Range of relation R is the set of all second elements of ordered pairs $(a,b) \in R$

$\therefore R_R = \{b : b \in B, (a,b) \in R\}$

(iii)The set B called co-domain of R.

13. **Function :** Let A and B are non-empty sets. A subset f of A X B is called a function from A to B iff $\forall$ x $\in A$, there exists a unique y $\in B$ such that $(x, y) \in f$. It is written $f : A \to B$.

Note : (a) The unique element y is called the image of the element x.

(b) The element x is called a pre-image of y.

(c) The set x is called the domain of function f.

(d) The set y is called the co-domain of function f.

(e) The set consisting of all the images of the elements of x under the function f called the range of f.

14. **One – One Function or Injective function :** A function $f : A \to B$ is said to be one-one (or injective), if the images of distinct elements of A under the function f are distinct.

15. **Many-One Function :** A function which is not one-one is called many-one function.

16. **Onto function or Surjective function :** A function $f : A \to B$ is said to be an onto function, if every element of the set B is an image of some element of set A under 'f'.

Note : The function $f : A \to B$ is onto if and only if range of $f = B$.

17. **Bijective function :** A function $f : A \to B$ is said to one-one and onto (or bijective) if it is both one-one and onto.

18. **Composition of function :** Let $f : A \to B$ and $g : B \to C$ be two functions. Then the composition of f and g, devoted by gof, is defined as the function $gof : A \to C$ given by $(gof)(x) = g(f(x)) \forall x \in A$.

Note : For gof to be defined, the range of f must be a subset of the domain of g.

19. **Invertible functions :** A function $f : A \to B$ is said to invertible if there exists a function $g : B \to A$ such that $gof = I_A$ and $fog = I_B$ where I_A and I_B are identity functions. The function g is called the inverse of f and is denoted by f^{-1}.

Note : If f is invertible, then f must be one – one and onto i.e. bijective and conversely, if f is one-one and onto, then f must be invertible.

20. **Binary Operation :** Let A and B are two non-empty sets. Then a function $f : A\ X\ A \to A$ is called a binary operation (or composition) on A.

Note : A binary operation is generally denoted by symbol $*, \Delta, O, +, X, ..etc$

The number of binary operations on set A having n elements $= (n(A))^{n(AXA)} = n^{n^2}$

21. **Properties of Binary Operations:** Let $*: A X A \to A$ be a binary operation on set A.

(i) Closed : A is said to be closed under the operation * if and only if $a, b \in A$

$\Rightarrow a * b \in A$

(ii) Commutativity : A binary operation * on the set A is said to be commutative, if $a * b = b * a \ \forall\ a, b \in A$

(iii) Associativity : A binary operation * on the set A is said to be associative, if $(a*b)*c = a*(b*c)\ \forall\ a,b,c \in A$.

(iv) Identity : An element $e \in A$ is said to be an identity element if and only if $e*a = a = a * e\ \forall\ a \in A$.

(v) Inverse of an element : An element $a \in A$ is said to be invertible if and only if there exists some $b \in A$ such that $a * b = e = b * a$.

So b is called inverse of a .

22. **Composition table :** It is a square array which indicates all the possible products. The elements to the left of the operation are written to the left vertical column of the array and the elements to the right of the operation are written on the top horizontal row. The intersecting boxes are filled by the following rule.

Element in the i^{th} row and j^{th} column = $a_i * a_j$

where a_i is the element on the left vertical column

a_j is the element on the top horizontal row.

23. **Theorem 1**. If R is an equivalence relation on set A then R^{-1} is also an equivalence relation.

Theorem 2. If $f : A \to B$, $g : B \to C$ and $h : C \to D$ are functions then $ho(gof) = (hog)of$.

Theorem 3. Let $f : A \to B$, $g : B \to C$ be two invertible functions. Then gof is also invertible with $(gof)^{-1} = f^{-1}og^{-1}$

24. Important properties :

(a) Any subset R of A X A is a relation on A.

(b) If R is a relation on set A, then R is symmetric iff $R^{-1} = R$.

(c) Empty set ϕ and A X A are two extreme relations and are also called **trivial solutions.**

(d) $f : A \to B$, is onto if and only if range of $f = B$.

(e) $f : A \to B$, is one-one if and only if

$$x_1 \neq x_2 \text{ in A} \Rightarrow f(x_1) \neq f(x_2) \text{ in B}$$

(f) $f:A\to B$ is invertible if and only if is both one-one and onto.

(g) The number of functions that can be defined as $f:A\to B$ is $[\text{n(B)}]^{n(A)}$.

(h) The number of one-one functions defined as $f:A\to A$ is n!

where n is the number of elements present in A.

(i) The number of one-one functions defined as $f:A\to B$ is P (m, n) i.e.

$$^{m}P_{n}=\frac{m!}{(m-n)!}$$

where n is the number of elements present in A and m is the number of elements present in B and n<m.

(j) Composite functions are not commutative.

(k) Composite functions are associative.

(l) The composition of a function with an identity function is the function itself.

(m) The composition of one-one function is also one-one.

(n) The composition of two onto functions is onto.

(o) In a composite function gof, if f and g are both one-one, then f and g need not necessarily be both one-one.

(p) If the composition of gof of two functions is onto, then both f and g need not be onto.

(q) gof is one-one implies that f is one-one.

(r) gof is onto implies that g is onto.

(s) If $f:A\to B$ is a function such that there exists a function $g:B\to A$ Such that $gof=I_A$ and $fog=I_B$, then f must be one-one and onto.

(t) The inverse of a function, if it exists, is unique

(u) If $f:A\to B$ is one-one and onto, then f^{-1} is also one-one and onto.

(v) The inverse of the inverse of a function is the function itself. i.e $(f^{-1})^{-1}=f$

(w) If $f:A\to B$ and $B\to C$ are two one-one and functions, then the inverse of gof exists and $(gof)^{-1}=f^{-1}og^{-1}$

(x) The identity f a binary operation, if it exits, is unique.

(y) In a binary operation, it is not necessary that each $a\in A$ has inverse $a^{-1}\in A$.

(z) $a_1\neq a_2$ does not necessarily mean that $a_1^{-1}\neq a_2^{-1}$.

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