IIT-JEE / ENGINEERING

MATHS

FORMULA

Inverse Trigonometric Functions

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IMPORTANT DEFINATIONS, FORMULAE AND METHODS

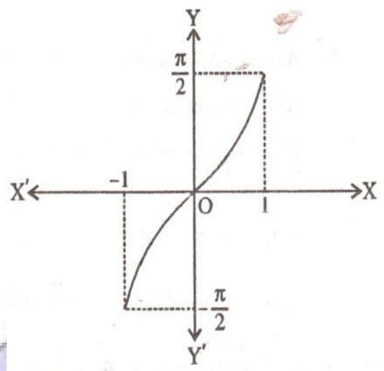
1. Arc sine function or Inverse sine function :

(i) Inverse sine function is written as $\sin^{-1}x$.

(ii) Domain of $\sin^{-1}x$ is [-1, 1]

(iii) Range of $\sin^{-1}x$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

(iv) The graph of $\sin^{-1}x$ is shown in fig.



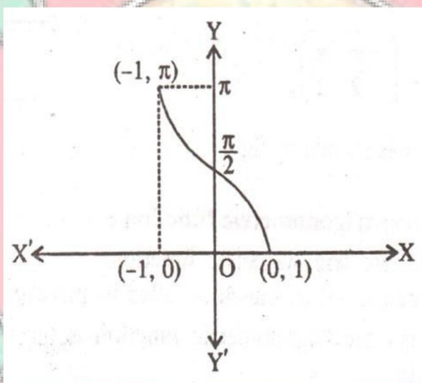
2. Arc cosine function or Inverse cosine function:

(i) Inverse cosine function is written as $\cos^{-1}x$.

(ii) Domain of $\cos^{-1}x$ is [-1, 1].

(iii) Range of $\cos^{-1}x$ is [0, π].

(iv) The graph of $\cos^{-1}x$ is shown in fig.



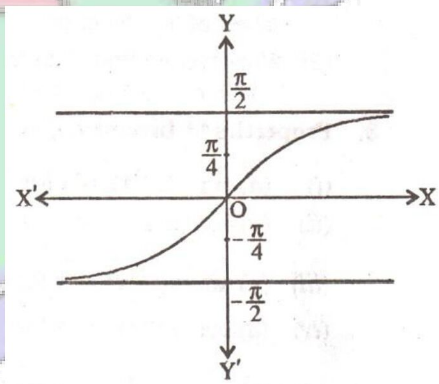
3. Arc tangent function or Inverse tangent function :

(i) Inverse tangent function is written as $\tan^{-1}x$.

(ii) Domain of $\tan^{-1}x$ is R.

(iii) Range of $\tan^{-1}x$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

(iv) The graph of $\tan^{-1}x$ is shown in fig.



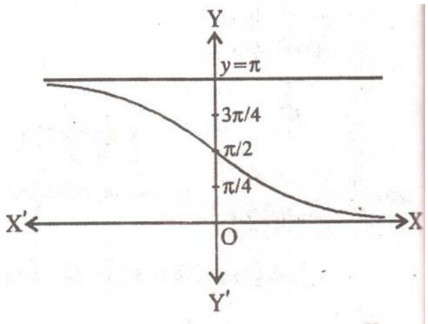
4. Arc co-tangent function or Inverse co-tangent function:

(i) Inverse cotangent function is written as $\cot^{-1}x$.

(ii) Domain of $\cot^{-1}x$ is R.

(iii) Range of $\cot^{-1}x$ is $(0, \pi)$.

(iv) The graph of $\cot^{-1}x$ is shown in fig.



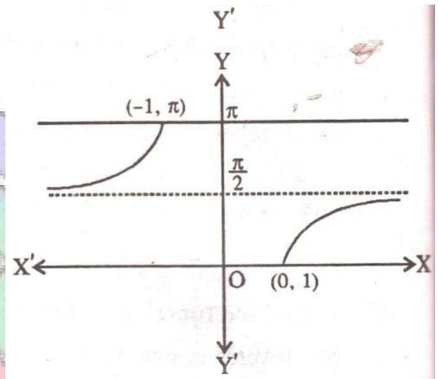
5. Arc secant function or Inverse secant function :

(i) Inverse secant function is written as $\sec^{-1}x$.

(ii) Domain of $\sec^{-1}x$ is R –(-1,1).

(iii) Range of $\sec^{-1}x$ is $[0,\pi] - \left\{\frac{\pi}{2}\right\}$.

(iv) The graph of $\sec^{-1}x$ is shown in fig.



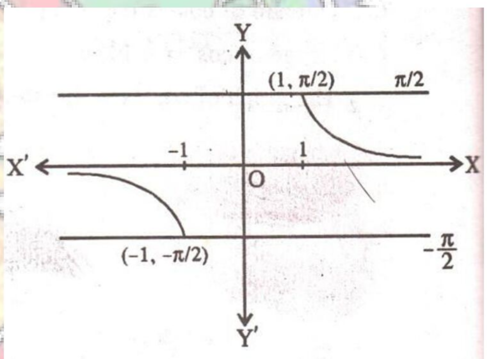
6. Arc cosecant function or Inverse cosecant function :

(i) Inverse cosecant function is written as $\text{cosec}^{-1}x$.

(ii) Domain of $\text{cosec}^{-1}x$ is R – (-1,1)

(iii) Range of $\text{cosec}^{-1}x$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$.

(iv) The graph of $\text{cosec}^{-1}x$ is shown in fig.



7. Principle Value of an inverse trigonometric functions :

(i) The value of an inverse trigonometric functions which lies in its principal value branch is called its principal value.

(ii) Principle value of an inverse trigonometric function is the least numerical value among all the values of that function.

(iii) Whenever no branch of an inverse trigonometric function is specifically mentioned the principal branch of the function should be taken.

8. Properties Value of Inverse trigonometric function :

(i) (a) $\sin(\sin^{-1}x) = x$ for $|x| \le 1$ (b) $\sin^{-1}(\sin\theta) = \theta$ for $|\theta| \le \frac{\pi}{2}$

(ii) (a) $\cos(\cos^{-1}x) = x$ for $|x| \le 1$ (b) $\cos^{-1}(\cos\theta) = \theta$ for $0 \le \theta \le \pi$

(iii) (a) $\tan(\tan^{-1}x) = x$ for $x \in R$ (b) $\tan^{-1}(\tan\theta) = \theta$ for $|\theta| < \frac{\pi}{2}$

(iv) (a) $\cot(\cot^{-1}x) = x$ for $x \in R$ (b) $\cot^{-1}(\cot\theta) = \theta$ for $0 < \theta < \pi$

(v) (a) $\sec(\sec^{-1}x) = x$ for $|x| \geq 1$ (b) $\sec^{-1}(\sec\theta) = \theta$ for $\theta \in [0,\pi] - \left\{\frac{\pi}{2}\right\}$

(vi) (a) $\text{cosec}(\text{cosec}^{-1}x) = x$ for $|x| \geq 1$

(b) $\text{cosec}^{-1}(\text{cosec}\,\theta) = \theta$ for $\theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$

(vii) (a) $\sin^{-1}\frac{1}{x} = \text{cosec}^{-1}x$ (b) $\cos^{-1}\frac{1}{x} = \sec^{-1}x$

(c) $\tan^{-1}\frac{1}{x} = \cot^{-1}x$

(viii) (a) $\sin^{-1}(-x) = -\sin^{-1}x$ (b) $\cos^{-1}(-x) = \pi - \cos^{-1}x$

(c) $\tan^{-1}(-x) = -\tan^{-1}x$ (d) $\cot^{-1}(-x) = \pi - \cot^{-1}x$

(e) $\sec^{-1}(-x) = \pi - \sec^{-1}x$ (f) $\text{cosec}^{-1}(-x) = -\text{cosec}^{-1}x$

(ix) (a) $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$ for $|x| \leq 1$ (b) $\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}$ for $x \in R$

(c) $\sec^{-1}x + \text{cosec}^{-1}x = \frac{\pi}{2}$ for $|x| \geq 1$

(x) (a) $\tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$, if $xy < 1$

(b) $$\tan^{-1}x + \tan^{-1}y = \begin{cases} \pi + \tan^{-1}\left(\frac{x+y}{1-xy}\right); x>0, y>0 \\ -\pi + \tan^{-1}\left(\frac{x+y}{1-xy}\right); x<0, y<0 \end{cases} : xy > 0$$

(c) $\tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x-y}{1+xy}\right)$; $x \geq 0, y \geq 0$

(d) $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \tan^{-1}\left(\frac{x+y+z-xyz}{1-xy-yz-zx}\right)$

(xi) $2\tan^{-1}x = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$, $|x| \leq 1$

(xii) $2\tan^{-1}x = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right), x \geq 0$

(xiii) $2\tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$, $|x| < 1$

(xiv) (a) $\sin^{-1}x + \sin^{-1}y = \sin^{-1}\left(x\sqrt{1-y^2} + y\sqrt{1-x^2}\right)$

(b) $\sin^{-1}x - \sin^{-1}y = \sin^{-1}\left(x\sqrt{1-y^2} - y\sqrt{1-x^2}\right)$

(xv) (a) $\cos^{-1}x + \cos^{-1}y = \cos^{-1}\left(xy - \sqrt{(1-x^2)(1-y^2)}\right)$

(b) $\cos^{-1}x - \cos^{-1}y = \cos^{-1}\left(xy + \sqrt{(1-x^2)(1-y^2)}\right)$

Note : if any mistake on this, kindly inform on the mail id :
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