IIT-JEE / ENGINEERING

MATHS

FORMULA

Determinants

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IMPORTANT DEFINATIONS, FORMULAE AND METHODS

1. Determinant of a square matrix of order two.

Let $A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$ be a square matrix of order 2, then

$$\det(A) = |A| = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{21}a_{12}$$

2. Determinant of the square matrix of order three.

Let A= $\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ be a square matrix of order 3, then its determinant is given by $\det A = |A| = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$

$$= a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

$$= a_{11}(a_{22}a_{33} - a_{32}a_{23}) - a_{12}(a_{21}a_{33} - a_{31}a_{23}) + a_{13}(a_{21}a_{32} - a_{31}a_{22})$$

Note : (1) Expanding a determinant along any row or column gives same value.

(2) For easier calculation, expand the determinant along that row or column which contains maximum number of zeros.

3. Properties of Determinants.

(a) The value of the determinant remains unchanged if its rows and columns are interchanged.

(b) If any two rows (or columns) of a determinant are interchanged, then sign of determinant changes.

(c) If any two rows (or columns) of a determinant are identical (i.e. all corresponding elements are same), then the value of determinants is zero.

(d) If all the elements of any row (or column) of a matrix 'A' are zero, then $|A| = 0$

(e) If each elements of a row (or a column) of a determinant is multiplied by a constant K, then the value gets multiplied by K.

(f) If some or all elements of a row or column of a determinant are expressed as sum of two (or more) terms, then the determinant can be expressed as sum of two (or more) determinants.

(g) If, to each element of any row or column of a determinant, the equimultiples of corresponding elements of other row (or column) are added, then value of determinant remains the same.

4. **Area of Traingles :** Area of a triangles ABC whose vertices are $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ is given by

Area of triangle, $\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

Note : (a) Since area is a positive quantity , so always take the absolute value of determinant.

(b) If area of triangle is given, then use both positive and negative value of the determinant.

(c) Three points A, B and C are said to be collinear if area of ΔABC = 0

5. **Minor :** Minor of an element a_{ij} of a determinant is the determinant obtained by deleting its i^{th} row and j^{th} column in which element a_{ij} lies. It is denoted as M_{ij}.

Note : Minor of an element of a determinant of order n is a determinant of order n-1.

6. **Cofactors :** Cofactor of an element a_{ij} is donoted as A_{ij} and is given by $A_{ij} = (-1)^{i+j} M_{ij}$.

Note : (1) Δ = sum of the product of elements of any row (or column) with their corresponding cofactors.

(2) If elements of a row (or column) are multiplied with cofactors of any other row(or column), then their sum is zero.

7. **Singular Matrix :** A matrix A is said to be a singular matrix if det (A) = 0.
8. **Non-Singular Matrix :** A matrix A is said to be non-singular if det (A) $\neq 0$.
9. **Adjoint of a Matrix :** The adjoint of a matrix A is the transpose of a matrix of cofactors of elements of matrix A. It is written as adj (A).

Note : (1) A (adj A) = (adj A) A = $|A| I$

(2) If A and B are non-singular matrices of same order, then AB and BA are also non-singular matrices of the same order.

(3) For matrices A and B of same order $|AB| = |A||B|$

(4) If A is a square matrix of order n, then $|adj(A)| = |A|^{n-1}$.

(5) If A is a square matrix of order n, $|KA| = K^n |A|$, where K is any scalar.

(6) If A is an invertible matrix then $(adjA)' = (adjA')$

10. Inverse of a Square Matrix. Let $A = [a_{ij}]$ be a square matrix of order n. The inverse of a matrix A is denoted as A^{-1} and is given by

$$A^{-1} = \frac{1}{|A|} adjA$$

Note : (1) If A and B are invertible matrices of same order, then

$$(AB)^{-1} = B^{-1}A^{-1}$$

(2) If A and B are invertible square matrices of same order then

$$adj(AB) = (adjB)(adjA)$$

11. Consistent system : A system of equations is said to be consistent if its solution exists.

12. Inconsistent system: A system of equations is said to inconsistent if its solution does not exist.

13. Method to find solution of system of non-homogeneous linear equations.

Matrix Method

Step I : Express the given system of equations in the form of AX = B.

Step II : Check, Is $|A| \neq 0$.

(a) If yes $(i.e.\ |A| \neq 0)$, then given system is consistent and has unique solution, given by $X = A^{-1}B$.

(b) If no, $(i.e.\ |A| = 0)$, then find (adj A) B

Step III: Check, Is $(adjA)B \neq 0$

(a) If yes, $(i.e.(adjA)B \neq 0)$, then given system is inconsistent i.e. no solution.

(b) If no $(i.e.(adjA)B = 0)$, then given system is consistent and has infinitely many solutions.

Note : if any mistake on this, kindly inform on the mail id :

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