IIT-JEE / ENGINEERING

MATHS

FORMULA

Applications of Derivatives

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IMPORTANT DEFINATIONS, FORMULAE AND METHODS

1. Rate of change of quantities :

(a) If a quantity y varies with another quantity x, such that $y = f(x)$, then $\frac{dy}{dx}$ represents the rate of change of y w.r.t. x and $\left.\frac{dy}{dx}\right]_{x=x_0}$ represents the rate of change of y w.r.t. x at $x = x_0$.

(b) If two variables x and y varying w.r.t. another variables t, i.e., if $x = f(t)$ and y = g (t), then by chain Rule

$$\frac{dy}{dx} = \frac{dy}{dt} X \frac{dt}{dx}$$

Thus the rate of change of y w.r.t. x can be calculated using the rate of change of y and that of x both w.r.t. t.

Here it should be noted that $\frac{dy}{dx}$ is positive if y increases as x increases and is negative if y decreases as x increases.

2. Increasing Function :

(a) Without using derivatives :

A function f is said to be increasing on an interval (a, b) if

$x_1 < x_2$ in $(a, b) \Rightarrow f(x_1) \le f(x_2)$ for all $x_1, x_2 \in$ (a, b).

(b) Using derivative :

A function f is increasing on (a, b) if $f'(x) \ge 0$ for each x in (a, b).

3. Decreasing Function :

(a) Without using derivatives :

A function f is decreasing on (a, b) if $x_1 < x_2$ in (a, b) $\Rightarrow f(x_1) \ge f(x_2)$ for all $x_1, x_2 \in (a, b)$.

(b) Using derivative :

A function f is decreasing on (a, b) if $f'(x) \le 0$ for each x in (a, b)

4. Strictly Increasing Function :

(a) Without using derivative :

A function f is strictly increasing on (a, b) if

$x_1 < x_2$ in $(a, b) \Rightarrow f(x_1) < f(x_2)$ for all $x_1, x_2 \in$ (a, b)

(b) Using derivative :

A function f is strictly increasing on (a, b) if $f'(x) > 0$ for each x in (a, b).

5. Strictly decreasing function

(a) Without using derivative :

A function f is strictly decreasing on (a, b) if $x_1 < x_2$ in $(a, b) \Rightarrow f(x_1) > f(x_2)$ for all $x_1, x_2 \in (a, b)$

(b) Using derivative :

A function f is strictly decreasing on (a, b) if $f'(x) < 0$ for each x in (a, b).

6. Critical Point :

A point on the curve $y = f(x)$ where either $f'(x)$ doesn't exist or $f'(x) = 0$ is called critical point.

7. Method to find the intervals in which a function is strictly increasing or strictly decreasing :

I. Let the function f is given by $f(x)$ on (a, b).

II. Find $f'(x)$ and using $f'(x) = 0$, find all the critical points satisfying the given interval (a, b). If interval is not mentioned then consider R i.e. $(-\infty, \infty)$ as the interval.

III. Arrange these critical points in ascending order. Let $x_1, x_2, x_3, x_4..........x_n$ be the critical points in (a, b) such that $x_1 < x_2 < x_3 < x_4......... < x_n$.

IV. Now check the sign of $f'(x)$ in the intervals $(a, x_1), (x_1, x_2), (x_2, x_3), (x_3, x_4)............(x_n, b)$.

V. The function f is strictly increasing on those intervals, in which $f'(x) > 0$ and strictly decreasing in which $f'(x) < 0$.

8. Slope of tangent to a curve :

Let $y = f(x)$ be a curve, then $m = \left.\frac{dy}{dx}\right|_{P(h,k)}$ is called slope of tangent to the given curve at point P(h, k).

9. Slope of Normal to a curve :

Let $y = f(x)$ be a curve and P(h, k) be a point on it. Slope of tangent to the given curve is $m = \left.\frac{dy}{dx}\right|_{P(h,k)}$

Therefore slope of normal to this curve at the same point is $-\frac{1}{m}$

10. Equation of tangent and normal to a curve :

Let P(h, k) be a point on the given curve $y = f(x)$ then equation of tangent to the given curve at P(h, k) is $y - k = m(x - h)$ and the equation of normal to this curve at the same point is $y - k = -\frac{1}{m}(x - h)$ where m is the slope of tangent to the given curve.

11. Important Note : Let m be the slope of tangent to a curve $y = f(x)$ at any point P(h, k) i.e. $m = \left.\frac{dy}{dx}\right|_{P(h,k)}$, then

I. $m = 0$, if the tangent is parallel to x – axis.

II. m is not defined $\frac{1}{m} = 0$, if the tangent is parallel to y-axis.

III. If the tangent makes an angle θ with positive x-axis, then $m = \tan\theta$.

IV. If the tangent is parallel to a line having slope m_1 then $m = m_1$.

V. If the tangent is perpendicular to a line having slope m_1, then m x m_1 = -1.

VI. Two curve touch each other if the slopes of their tangents are equal at the points of intersection of the curves.

VII. Two curves are orthogonal if $m_1 m_2 = -1$ where m_1 and m_2 be the slopes of their tangents at the points of intersection of the curves.

12. Differentials :

For a function $y = f(x)$

I. The differential of x, denoted by dx, is defined by $dx = \Delta x$.

II. The differential of y, denoted by dy , is defined as $dy = \left[\frac{dy}{dx}\right]\Delta x$.

13. Approximations :

We can use the differentials to approximate values of certain quantities.

Let $y = f(x)$ be a function, then Δy is the actual change in y corresponding to small increment Δx in x. If δx is very-very small as compared to x, then dy is nearly equal to Δy. dy is the approximate change in y, so when Δx is very very small as compared to x, we have $dy \approx \Delta y$.

14. Absolute Error :

The error Δx in x is called absolute error in x.

15. Relative Error :

$\frac{\Delta x}{x}$ is called relative error in x.

16. Percentage Error :

$\frac{\Delta x}{x}$ x 100 is called percentage error in x.

17. Maxima and Minima :

Let f be a function defined on an interval I. Then

(a) f is said to have a maximum value in I, if there exists a point c in I such that $f(c) \geq f(x)$, for all $x \in I$.

The number $f(c)$ is called the maximum value of f in I and the point c is called a point of maximum value of f in I.

(b) f is said to have a minimum value in I, If there exists a point c in I such that $f(c) \leq f(x)$, for all $x \in I$.

The number $f(c)$ is called the minimum value of f in I and the point c is called a point of minimum value of f in I.

18. Monotonic Function :

A function which is either increasing or decreasing in the given interval I is called monotonic function. Every monotonic function assumes its maximum/minimum value at the end points of the domain of definition of the function.

19. Local maxima and Local minima :

Let f be a function and Let c be an interior point in its domain, then

(a) c is called a point of local maxima if there is an h > 0 such that $f(c) \geq f(x)$, for all $x \in (c-h,\ c+h)$.

The number $f(c)$ is called the local maximum value of f.

(b) c is called a point of local minima if there exists an h > 0 such that $f(c) \leq f(x)$, for all $x \in (c-h,\ c+h)$.

The number $f(c)$ is called the local minimum value of f.

20. Method to find local maxima or Local minima using first or second derivative test :

I. Let the given function be $f(x)$.

II. Find $f'(x)$ and using $f'(x)=0$, find the critical points of $f(x)$ say $x = x_1, x_2$.......

III. Find $f''(x)$ and check the sign of $f''(x)$ at these critical points.

IV. Those points for which $f''(x) > 0$, are known as points of local minima and for those $f''(x) < 0$ are known as points of local maxima.

V. **Case of Failure :** If $f''(x) = 0$ at any critical point say $x = x_1$, then second derivative test fails.

Now we can use first derivate test to find local maxima or minima.

VI. **First derivation test :**

(a) First check the sign of $f'(x)$ when x is slightly less than x_1.

(b) Then secondly check the sign of $f'(x)$ when x is slightly greater than x_1.

(c) If $f'(x)$ changes its sign from + to - , then x_1 is the point of local maxima.

(d) If $f'(x)$ changes its sign from - to + , then x_1 is the point of local minima.

(e) If $f'(x)$ does not change its sign, then x_1 is called point of inflexion, means the point where there is no maxima or minima.

Note : if any mistake on this, kindly inform on the mail id :

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