IIT-JEE / ENGINEERING

MATHS

FORMULA

Differential Equations

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IMPORTANT DEFINATIONS, FORMULAE AND METHODS

1. **Differential Equation :** A differential equation is an equation which involves an independent variables, a dependent variable and the differential co-efficient.
2. **Ordinary Differential Equation :** A differential equation involving derivatives of the dependent variable with respect to only one independent variable is called an ordinary differential equation.
3. **Order of Differential Equation :** It is the order of the highest derivative appearing in the equation.
4. **Degree of Differential Equation :** It is the highest power (positive integral index) to which the highest order derivative is raised when the differential equation is written as a polynomial in the derivatives.
5. **Solution of Differential Equation :** It is the relationship between the variables (not involving their derivatives) which satisfies the given differential equation.
6. **General (or Complete) Solution :** It is the solution in which the number of independent arbitrary constants is equal to the order of the differential equation.
7. **Particular Solution :** It is the solution obtained from the general solution by giving particular value(s) to the arbitrary constant(s).
8. **Homogeneous Differential Equation :** A differential equation of the form $\frac{dy}{dx} = f(x, y)$ is said to be homogeneous if $f(x, y)$ is a homogeneous function of degree zero.
9. **Linear Differential Equation :** A differential equation of the form $\frac{dy}{dx} + Py = Q$, or $\frac{dx}{dy} + Px = Q$, where P and Q are constants or functions of x is known as first order linear differential equation.
10. **Procedure to form a Differential equation representing a family of curves depending on one parameter.**

    Let the family of curves be $f(x, y, a) = 0$ …(1)

    (i) Differentiate (1) with respect to x. Let new relation is $g(x, y, y', a) = 0$ …(2)

    (ii) Eliminate 'a' from equations (1) and (2) to get required differential equation.

11. Procedure to form a Differential equation representing a family of curves depending on two parameters.

Let the family of curves be $f(x, y, a, b) = 0$ ...(1)

(i) Differentiate (1) with respect to x. Let new relation is $g(x, y, y', a, b) = 0$ ...(2)

(ii) Differentiate (2) with respect to x. Let new relation is $h(x, y, y', y'', a, b) = 0$

...(3)

(iii) Eliminate a and b from equations (1), (2) and (3) to get required differential equation.

12. Procedure to solve Homogeneous Differential Equation :

(i)Let $\frac{dy}{dx} = \frac{f(x, y)}{g(x, y)}$ be a homogeneous differential equation. ....(1)

(ii) Put $y = Vx$ in equation (1) so that $\frac{dy}{dx} = V + x\frac{dV}{dx}$

(iii) Equation (1) will reduce in variable separable form.

Note : If homogeneous equation is in the form of $\frac{dx}{dy} = \frac{f(x, y)}{g(x, y)}$ then, put $x = Vy$ so that $\frac{dx}{dy} = V + y\frac{dV}{dy}$

13. Procedure to solve first order linear differential equation :

Consider the equation $\frac{dy}{dx} + Py = Q$.

(i)Find the Integrating factor $(I.F.) = e^{\int P dx}$.

(ii) Write the solution of given linear differential equation as

$$y(I.F.) = \int Q.\ (I.F.)dx + C$$

(iii)Solve the above integral to get required solution.

Note : If the linear differential equation is of the form $\frac{dx}{dy} + Px = Q$, then find $(I.F.) = e^{\int P dx}$ and write the solution is as $x.(I.F.) = \int Q.\ (I.F.)dy + C$.

14. Note :

(a) To find the degree of a differential equation, make sure that the differential equation must be a polynomial equation in derivatives.

(b) Order and degree (if defined) of a differential are always positive integers.

(c) The order of differential equation representing a family of curves is same as the number of arbitrary constants present in the equation corresponding to the family of curves.

Note : if any mistake on this, kindly inform on the mail id :

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